Math 371, Spring 2006 — About the Second Test

The second test for this course will take place in class on Wednesday, April 26. It will cover everything that we have done in class since the first test. This includes Sections 2.4, 2.5, 2.6, 2.7, 2.8, 2.10, 3.1, 3.2, 3.4, 3.5, 4.3 and 4.4 in Walker's *A Primer on Wavelets*, as well as other material that we have covered in class.

Of course, you should also be familiar with general ideas from the first part of the course, such as the general nature of Fourier analysis and wavelet analysis and how they compare.

Presentations of final projects will take place on Friday, April 28 and Monday, May 1. The written final project is due on Sunday, May 7, or before.

List of Topics for the Test

compression of signals

lossless versus lossy compression

noise removal

how compaction of energy relates to compression and noise removal

thresholding

significance map

quantization

numerical measures of the effectiveness of compression and noise removal

root mean square error, RMS Error $= \sqrt{\frac{1}{N}\left((f_1 - s_1)^2 + \cdots + (f_N - s_N)^2\right)} = \frac{\|\mathbf{f}-\mathbf{s}\|}{\sqrt{N}}$

$\mathbb{R}^{nm}$ considered as the vector space of $n \times m$ matrices

tensor product, $\mathbf{x} \otimes \mathbf{y} \in \mathbb{R}^{nm}$, of two vectors $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$

$(a\mathbf{x} + b\mathbf{y}) \otimes \mathbf{z} = a(\mathbf{x} \otimes \mathbf{z}) + b(\mathbf{y} \otimes \mathbf{z})$ and $\mathbf{x} \otimes (a\mathbf{y} + b\mathbf{z}) = a(\mathbf{x} \otimes \mathbf{y}) + b(\mathbf{x} \otimes \mathbf{z})$

basis of $\mathbb{R}^{nm}$ of the form $\{\mathbf{x}_i \otimes \mathbf{y}_j \,|\, 1 \le i \le n, 1 \le j \le m\}$ for bases $\{\mathbf{x}_i\}$ of $\mathbb{R}^n$ and $\{\mathbf{y}_j\}$ of $\mathbb{R}^m$

bilinear functions on $\mathbb{R}^n \times \mathbb{R}^m$ and their relation to linear functions on the tensor product space $\mathbb{R}^{nm}$

images; images considered as two-dimensional signals

discrete two-dimensional scaling functions and wavelets

2D wavelet transform of a matrix: a^1, h^1, v^1, and d^1

multi-level 2D wavelet transforms of a matrix

2D wavelet transformation computed using the basis of tensor products

2D wavelet transformation computed by applying 1D transform to rows and then to columns of a matrix

computing 2D Haar transforms of a matrix

compression of images

compressing the significance map; zero-trees

noise removal from images

DFT (discrete Fourier transform): $(\mathcal{F}\mathbf{f})_k = \sum_{n=0}^{N-1} f_n e^{-2\pi i k n/N}$

FFT (fast Fourier transform), and how it compares in speed to the regular DFT

convolution of discrete signals: $(\mathbf{f} \star \mathbf{g})_k = \sum_{j=0}^{N-1} f_j g_{k-j}$

convolution theorem: $\mathcal{F}(\mathbf{f} \star \mathbf{g}) = (\mathcal{F}\mathbf{f})(\mathcal{F}\mathbf{g})$

using the convolution theorem and the FFT to quickly compute convolutions of discrete signals

the two-dimensional DFT

computing the 2D DFT by applying the 1D DFT to the rows of a matrix then to the columns

the two-dimensional FFT, and how it compares in speed to computing the 2D DDT directly

L_2

scaling function $\varphi(x)$

$\varphi_{j,k}(x) = \frac{1}{\sqrt{2^j}}\,\varphi\big(\frac{x}{2^j} - k\big)$

the filter coefficients h_n, satisfying: $\varphi(x) = \sum_{n=-\infty}^{\infty} h_n \sqrt{2}\,\varphi(2x-n)$ and therefore $\varphi_{j+1,k}(x) = \sum_{n=-\infty}^{\infty} h_n \varphi_{j,2k+n}(x)$

the multiresolution $\ldots, V_{-2}, V_{-1}, V_0, V_1, V_2, \ldots$, where V_j is the space generated by $\{\varphi_{j,k} \mid k \in \mathbb{Z}\}$

the approximation operators $\mathcal{P}_j(f) = \sum_{k=-\infty}^{\infty} \langle f, \varphi_{j,k}\rangle \varphi_{j,k}$

the wavelet $\psi(x) = \sum_{n=-\infty}^{\infty} g_n \sqrt{2}\,\varphi(2x-n)$, where $g_n = (-1)^{1-n} h_{1-n}$

wavelets encode the differences between one approximation and the next: $\mathcal{P}_{j-1}(f) - \mathcal{P}_j(f) = \sum_{k=-\infty}^{\infty} \langle f, \psi_{j,k}\rangle \psi_{j,k}$

orthonormal basis for L_2 consisting of wavelets $\{\psi_{j,k} \mid j, k \in \mathbb{Z}\}$

general idea of how scaling functions, wavelets, and multiresolutions can be consrtucted from $\hat{h}(\xi)$

the discrete wavelet transform uses the filter coefficients h_n and g_n, not the scaling functions and wavelets!

continuous wavelet transforms and scalograms (Sections 4.2 and 4.3 of Walker)

continuous wavelet transform defined in terms of convolution

how to compute scalograms of discrete signals efficiently using the FFT and the convolution theorem