

The second test for this course will be given in class on Wednesday, October 21. It will cover mainly Chapters 5, 6, and 7. However, it is important for you to know material from earlier chapters as well. Furthermore, you should be familiar with the fundamentals of equivalence relations.

Here are some suggested practice problems. Most of these are questions that could be given on a test. A few are too long or tricky to be appropriate for a test. Unfortunately, most of the Supplementary Exercises for Chapters 5 through 8 seem to require material from Chapter 8, so only a few exercises from that set are appropriate:

- Chapter 5, p. 111, # 3, 9, 17, 2, 27, 45, 51
- Chapter 6, p. 129, # 3, 21, 23, 33
- Chapter 7, p. 145, # 7, 9, 13, 17, 27, 35
- Supplementary Exercises, p. 169, # 1, 6, 7, 8, 23, 29, 41, 49

Here are some important things you should know about, including some stuff from earlier chapters

$\gcd(a, b)$, $\text{lcm}(a, b)$, $a \bmod n$

subgroup

Abelian group

the center, $Z(G)$, of a group

Important groups: \mathbf{Q} , \mathbf{Q}^+ , \mathbf{Q}^* , \mathbf{Z} , \mathbf{Z}_n , $U(n)$, D_n , S_n , A_n

cyclic group

A finite cyclic group of order n has exactly one subgroup for each divisor of n .

The number of elements of order d in a finite group is a multiple of $\varphi(d)$.

permutation groups

cycles

the order of product of *disjoint* cycles

Every permutation of a finite set can be written as a product of 2-cycles.

even and odd permutations

group isomorphisms and automorphisms

the groups $\text{Aut}(G)$ and $\text{Inn}(G)$

$\text{Aut}(\mathbf{Z}_n)$ is isomorphic to $U(n)$

equivalence relations

reflexive, symmetric, and transitive properties of a relation

equivalence classes

The equivalence classes of an equivalence relation on a set X partition X .

cosets

left cosets (aH) and right cosets (Ha)

aHa^{-1}

If H is a subgroup of G , the cosets aH partition G .

If H is a subgroup of G , all cosets aH have the same number of elements.

Lagrange's Theorem: If H is a subgroup of a finite group G , $|H|$ divides $|G|$.

If a is an element of a finite group $|G|$, then $|a|$ divides $|G|$.

Fermat's Little Theorem: $a^p \bmod p = a \bmod p$, for a prime p and $a \in \mathbf{Z}$.

Every group of prime order p is cyclic.

Every group of order $2p$, where p is prime, is isomorphic to \mathbf{Z}_{2p} or to D_p .