The second test for this course will be given in class on Wednesday, October 26. It will cover Chapters 5, 6, 7, and 8 from the textbook. Of course, you need to know the material from earlier chapters, but there will be no questions specifically directed at that material. In addition, Chapter 7 is really just background material on functions; there will be no questions about abstract functions as such, but there might be questions about functions as they relate to group theory such as proving that a group-theoretical function is one-to-one and/or onto.

There will be some proofs on the test, but they will be proofs that are reasonably straightforward and that should not require a lot of thought, as long as you know the material. There will be some short answer questions to test your knowledge of definitions and theorems. And there will be a few computational questions, such as finding the order of a permutation or. It is possible that there will be a longer essay-type question.

Here are some important things you should know about:

Subgroup

Proving that a subset is a subgroup

$Z(G)$, the center of a group G

$Z(g)$, the centralizer of an element g in a group G

Subgroups of GL(2,$\mathbb{R}$): upper triangular matrices; SL(2,$\mathbb{R}$)

The intersection of subgroups is a subgroup

The union of two subgroups, $H \cup K$, is a subgroup if and only if $H \subseteq K$ or $K \subseteq H$

Every subgroup of a cyclic group is cyclic

If $G = \langle x \rangle$ is a finite cyclic group, and H is a subgroup, then $|H|$ divides $|G|$

If $G = \langle x \rangle$ is a finite cyclic group, then G has a unique subgroup for every divisor of $|G|$

If $G = \langle x \rangle$ is a finite cyclic group of order n, and $k \in \mathbb{Z}^+$, then $\langle x^k \rangle = \langle x^{\gcd(n,k)} \rangle$

Finding all subgroups of $\mathbb{Z}_n$

The Euler phi-function $\phi(n)$

The number of elements of order d in a finite cyclic group of order n is 0 if $d \nmid n$ and is $\phi(d)$ if $d \mid n$.

Direct product of groups

If $G_1, G_2, \ldots, G_n$ are any groups and $(g_1, g_2, \ldots, g_n) \in G_1 \times G_2 \times \cdots \times G_n$ then $o\big((g_1, g_2, \ldots, g_n)\big) = \infty$ iff $o(g_i) = \infty$ for some i

If $G_1, G_2, \ldots, G_n$ are any groups and $(g_1, g_2, \ldots, g_n) \in G_1 \times G_2 \times \cdots \times G_n$ where each $o(g_i) < \infty$, then $o\big((g_1, g_2, \ldots, g_n)\big) = \operatorname{lcm}\big(o(g_1), o(g_2), \ldots, o(g_n)\big)$

If $G_1, G_2, \ldots, G_n$ are cyclic groups, then $G_1 \times G_2 \times \cdots \times G_n$ is cyclic iff $\gcd(|G_i|, |G_j|) = 1$ for all $i \neq j$

$\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \cdots \times \mathbb{Z}_{n_k}$ is cyclic iff $\gcd(n_i, n_j) = 1$ for all $i \neq j$

Every element of $\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \cdots \times \mathbb{Z}_{n_k}$ has order that divides $o\big((1, 1, \ldots, 1)\big)$

$G_1 \times G_2 \times \cdots \times G_n$ is abelian iff G_i is abelian for all i

One-to-one function

Onto function

Bijection

Inverse function

Permutation of a set X

The symmetric group $(S_X, \circ)$ of permutations of X under the operation of function composition

The identity function, as the identity of the group S_X

The group S_n, equal to S_X where $X = \{1, 2, 3, \ldots n\}$

Tableau notation for permutations in S_n

Computing products in S_n

Cycle; length of a cycle

For a cycle $\alpha = (x_1, x_2, \ldots, x_r)$, $o(\alpha) = r$

Every permutation can be written as a product of disjoint cycles of length ≥ 2, in a unique way up to order

Disjoint cycles commute

If α_1, α_2, $\ldots$, α_k are disjoint cycles, then $o(\alpha_1 \alpha_2 \cdots \alpha_k) = \operatorname{lcm}\big(o(\alpha_1), o(\alpha_2), \ldots, o(\alpha_k)\big)$

Every permutation can be written as a product of transpositions

Even and odd permutations

Lemma: The identity permutation cannot be written as a product of an odd number of transpositions

No permutation can be both even and odd; the proof of this fact from the Lemma

The alternating group A_n

$|S_n| = n!$ and $|A_n| = \frac{n!}{2}$

A_4 is isomorphic to the group of rotational symmetries of the tetrahedron

The dihedral group D_n as a subgroup of S_n

The generators f and g of D_n

$D_n = \{e, f, f^2, \ldots, f^{n-1}, g, gf, gf^2, \ldots, gf^{n-1}\}$