This homework is due Wednesday, September 24

1. Find all complex numbers z such that

a) $e^z = -1$

b) $e^z = 1 - i\sqrt{3}$

c) $\cos(z) = 2$. [Hint: Rewrite the equation as $(e^{iz})^2 + 1 = 4e^{iz}$, then let $w = e^{iz}$. Expect the answer to be ugly.]

2. Show by direct calculation using the definitions that $\sin^2(z) + \cos^2(z) = 1$ and that $\sin(z+w) = \sin(z)\cos(w) + \cos(z)\sin(w)$ for all z.

3. Recall that the hyperbolic functions $\sinh(x)$ and $\cosh(x)$ are defined for real x by

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

The same definition can be used when the argument is complex.

a) How can sinh and cosh can be easily expressed in terms of sin and cos?

b) Use the results from **a)** and a formula from exercise **2** to find a formula relating $\sinh^2(z)$ and $\cosh^2(z)$.

c) Use the results from **a)** and a formula from exercise **2** to find a formula for $\sinh(z + w)$.

4. (Parts a and b are 3.8 and 3.9 from the textbook.)

a) Find all analytic functions $f = u + iv$ with $u(x, y) = x^2 - y^2$.

b) Show that there is no analytic function $f = u + iv$ with $u(x, y) = x^2 + y^2$.

c) Consider a function $u(x, y)$. Use the Cauchy-Riemann functions to find a necessary condition on the *second* partial derivatives of u that must be satisfied if u is the real part of a complex analytic function. How does your condition apply to **a)** and **b)**?

5. Compute **(a)** $\displaystyle\int_0^1 t + i\sqrt{t}\,dt$ and **(b)** $\displaystyle\int_0^{\pi/4} e^{it} dt$

6. Using the definition, find the value of $\displaystyle\int_C z^2 dz$, where C is the curve given by $z(t) = t + it^2$ for $t \in [0, 2]$,

7. Individual assignments: Each person will present one of the following short proofs from the book. We will decide who does what on Friday. The presentations should be careful and complete, filling in details of the proofs from the book if necessary. The presentations will probably be on Wednesday, when the homework is due.

a) Propositions 3.6 and 3.7, page 39.

b) Definition 4.6 and Proposition 4.7, page 47. (There is a missing minus sign in the proposition.)

c) Lemma 4.9, page 49.

d) Proposition 4.11, page 50.