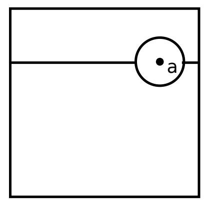
Proposed Schedule

Wednesday, October 15: Study guide for test is distributed.
Friday, October 17: This homework is due; take-home test is distributed.
Monday, October 20: In-class part of the test.
Wednesday, October 22: Take-home part of the test is collected.

1. The Cauchy Integral Formula is stated for integrals along a circle. Let S be a square, and let Γ be its boundary traced in the counterclockwise direction. Let f be a function that is analytic on S, and a a point in the interior of S. Show that $f(a) = \frac{1}{2\pi i}\int_\Gamma \frac{f(z)}{z-a}\,dz$. (Hint: Consider the following picture.)



Note: This is a preview of a more general form of the Cauchy Integral Formula.

2. Find the values of the following integrals. Almost no computation is needed, but you should clearly justify your answers.

a) $\frac{1}{2\pi i}\int_C \frac{e^z}{z-2}\,dz$, where C is the circle $|z| = 3$

b) $\frac{1}{2\pi i}\int_C \frac{e^z}{z-2}\,dz$, where C is the circle $|z| = 1$

c) $\int_C \frac{z^3 - 3z + 5}{z - i}\,dz$, where C is the circle $|z| = 2$

d) $\int_C \frac{e^{\pi z}}{z^4}\,dz$, where C is the circle $|z| = r$ for any $r > 0$.

3. Suppose that f is analytic on $D(0;1)$ and that $f(\frac{1}{n}) = \frac{1}{n^2}$ for $n \in \mathbb{Z}^+$. What can you say about f?

4. Suppose that $f(z)$ is an entire function such that $Re(f(z))$ has an upper bound. That is, there exists $M \in \mathbb{R}$ such that $Re(f(z)) < M$ for all z. Show that f is constant. (Hint: Consider the function $e^{f(z)}$.) Also, by considering $-f(z)$, show that if $Re(f(z))$ has a lower bound, then f is constant. What can you say about $Im(f(z))$, and why?

5. Suppose that $f(z)$ is a non-constant analytic function on a bounded domain D and that f is continuous on the closure $\tilde{D}$. Since $\tilde{D}$ is compact, we know that $Re(f(z))$ has a minimum and a maximum value on $\tilde{D}$. Show that the minimum and maximum must occur on the boundary of $\tilde{D}$ and never at interior points.