This homework is due Friday, November 10

For all problems, show all steps in the required computation!

1. Compute the residue of each of the following functions at each singularity:

a) $\dfrac{z - \sin(z)}{z^4}$

b) $\left(\dfrac{z}{2z+1}\right)^3$

c) $\dfrac{z^2+1}{(z-1)(z^2+4)}$

2. Let C be the circle $|z| = 2$. Evaluate $\displaystyle\int_C \frac{dz}{z^3(z+4)}$

3. Let C be the unit circle. Evaluate $\displaystyle\int_C z^n e^{1/z} dz$ for positive integers n.

4. (a) Show that the only zeros of $\cos z$ in $\mathbb{C}$ are the real numbers $z = \frac{\pi}{2} + n\pi$, $n \in \mathbb{R}$.

(b) Evaluate the integral $\displaystyle\int_C \frac{\cos(z)}{\cos(2z)}\, dz$, where C is the unit circle.

5. Suppose that $f(z)$ is analytic on and inside the unit circle. Let $\lambda \in \mathbb{C}$ such that $|\lambda| < 1$. Use Rouché's Theoem to show that $f(z)$ and $(1 + \lambda z)f(z)$ have the same number of zeros inside the unit circle (counting multiplicity).

About your final presentation: As you know, there is no final exam for this course. However, there is a final presentation. The presentations will take place during the regularly scheduled final exam period for the course, on Thursday, December 18, at 1:30 PM.

Your presentation should be about 30 minutes long, allowing some time at the end for questions. It should be well-prepared and polished. In fact, you should plan to meet with me during the final week of classes or during reading period to go through a practice run of the presentation.

You should meet with me soon to discuss possible topics. Part of the next homework assignment will be to turn in a statement of your presentation topic and a possible outline for the presentation.

You are not being asked to do anything original in the presentation. The topic can be something from the textbook, or it can be something from outside the class related to complex analysis. Topics from the book can be somthing we skipped, some important result that we covered, or something that comes later in the book. Here are some ideas:

- Cover some of the material in Section 5.3 on contractions and Newton's Method.
- Cover Schwartz's Lemma (7.2, page 94) and some of its consequences.
- Prove the Rectangle Theorem (4.14).
- Prove Cauchy's Integral Formula.
- Show that every complex differentiable function is infintely differentiable.
- Present some applications of the Residue Theorem to real integrals.
- Discuss analytic continuation (see Chapter 18).
- Derive the solution of the general cubic polynomial and explain its significance for complex numbers.
- Discuss the application of complex numbers in quantum theory.