The test will have an in-class part and a take-home part. The in-class part of the test takes place on Monday, October 20. It can include computational questions, statements of theorems, definitions, longer essay-type questions about concepts, and perhaps some simple proofs. The test covers Chapters 1 through 6 in the textbook. However, we did not cover the solution of the cubic, Newton's Method, or saddle points.

The take-home part is scheduled to be distributed on Friday, October 17 and collected on Wednesday, October 22. However, we can discuss the schedule in class on October 15. The take-home part will consist mostly of more complex computational questions and proofs.

Terms and ideas that you should know:

the complex numbers $\mathbb{C} = \{a + ib \mid a, b \in \mathbb{R}\}$

arithmetic of complex numbers: addition, subtraction, multiplication, division

real and imaginary parts of a complex number z: $\text{Re}(z)$ and $\text{Im}(z)$

complex conguate $\overline{z}$

modulus or absolute value of a complex number, $|z|$

polar form of a complex number $z = r(\cos\theta + i\sin\theta)$

argument of a complex number, $\text{Arg}(z)$

the complex plane

geometric meaning of complex addition and subtraction

geometric meaning of complex multipilication, for complex numbers in polar form

important properties of complex numbers, including:

$|zw| = |z| \cdot |w|$

$|z + w| \leq |z| + |w|$

$z\overline{z} = |z|^2$

topogical aspects of the complex numbers, including

open neigborhood of radius r about a point z: $D(z; r)$

open set, limit point of a set, boundary of a set, closed set

connected set

region: a connected open set

continuous function

sequences; limit of a sequence

series; partial sums of a series; convergent series; sum of a series

power series $\displaystyle\sum_{k=0}^{\infty} c_k z^k$ and $\displaystyle\sum_{k=0}^{\infty} c_k (z-a)^k$

the "lim sup" of a sequence of real numbers: $\overline{\lim\limits_{n\to\infty}}\, a_k$

radius of convergence of a series

the derivative of a power series; powers series define infinitely differentiable functions

the derivative of a complex function as $f'(z) = \lim\limits_{h\to 0} \dfrac{f(z+h) - f(z)}{h}$

the Cauchy-Riemann equations: $u_x = v_y$ and $u_y = -v_x$

for a differentiable function, $f' = u_x + iv_x$

analytic function (analytic at z means differentiable on an open neighborhood of z)

entire function

integration of complex functions: $\displaystyle\int_a^b z(t)\,dt = \int_a^b x(t)\,dt + i\int_a^b y(t)\,dt$

smooth paths $z\colon [a,b] \to \mathbb{C}$ (really should be called piecewise smooth)

line integrals $\displaystyle\int_C f(z)\,dz = \int_a^b f(z(t))z'(t)\,dt$

line integrals are independent of parameterization (as long as the direction is the same)

closed curves and simple closed curves

important properties of line integrals, including

$$\int_{-C} f(z)\,dz = -\int_C f(z)\,dz$$

$$\int_{C_1+C_2} f(z)\,dz = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz$$

$$\left|\int_C f(z)\,dz\right| \le ML, \text{ if } L \text{ is the length of } C \text{ and } |f(z)| < M \text{ on } C$$

an entire function is given by a power series that converges everywhere

a function analyic on $D(a;r)$ is given by a power series about a that converges on that disk

analytic functions are infinitely differentiable

compact subsets of $\mathbb{C}$ (closed and bounded)

accumulation point of a set

the entire functions e^z, $\sin(z)$, and $\cos(z)$

$e^z = e^x(\cos(y) + i\sin(y))$, where $z = x + iy$; $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ for $\theta \in \mathbb{R}$

the geometric effect of the exponential function

the power series for e^z, $\displaystyle e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}$

the multi-valued complex logarithm, $\mathrm{Log}(z) = \ln(|z|) + i\mathrm{Arg}(z)$

the geometric series, $\displaystyle\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k$, for $|z| < 1$

Important Theorems:

Root Test for Power Series: Consider a power series $\displaystyle\sum_{k=0}^{\infty} c_k(z-a)^k$. Let $L = \overline{\lim_{k\to\infty}}\, |c_k|$, and let $R = 1/L$, If $L = 0$, then the series converges for all z. If $L = \infty$, then the series converges only for $z = a$. If $0 < L < \infty$, then the series converges for $|z| < R$, and the series diverges for $|z| > R$.

Cauchy-Riemann Equations and Analytic Functions: Suppose that f is defined on an open set D, that the partial derivatives of u and v exist and are continuous, and that they satisy the Cauchy-Riemann equations on D. Then $f'(z)$ exists for $z \in D$.

Complex analog of the Fundamental Theorem of Calculus: Suppose C is a smooth curve, given by $z\colon [a,b] \to \mathbb{C}$. Suppose that $F(z)$ is analytic on C, and that $f(z) = F'(z)$. Then

$\int_C f(z)\,dz = F(z(b)) - F(z(a))$. In particular, if C is a closed curve, then $\int_C f(z)\,dz = 0$,

Rectangle Theorem: If Γ is the boundary of a rectangle R and if $f(z)$ is analytic on an open set containing R, then $\int_\Gamma f(z)\,dz = 0$,

Integral Theorem: If $f(z)$ is analytic on an open disk $D(a;r)$ (possibly of infinite radius) and if $F(z)$ is defined by $F(z) = \int_\Gamma f(z)\,dz$, where Γ is the curve from a to z consisting of a horizontal followed by a vertical line segment, then F is analytic and $F'(z) = f(z)$ on $D(a;r)$.

Path Independence: If f is entire and C_1 and C_2 are both smooth curves from z_1 to z_2, then $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$. The common value of these integrals can then be written $\int_{z_1}^{z_2} f(z)\,dz$. (In fact, path independence is true if f is analytic on a disk that contains C_1 and C_2. However, it is not true for analytic functions on arbitrary open sets.)

Cauchy Integral Formula: Suppose $f(z)$ is analytic on a disk $D(z_o;R)$ (possibly of infinite radius), that $0 < r < R$, and that $|a - z_0| < r$ so that a is inside the circle of radius r about z_o. Let C be the circle of radius r about z_o. Then $f(z) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$.

Taylor Series for an Analytic function: Suppse that $f(z)$ is analytic on $D(a;r)$ (possibly of infinite radius). Then $f^{(k)}(a)$ exists for all $k = 0, 1, 2, \ldots$, and $f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(z-a)^k$ for $z \in D(a;r)$.

Liouville's Theorem: A bounded entire function is constant.

Fundamental Theorem of Algebra: Every non-constant polynomial has a root in $\mathbb{C}$.

Uniqueness Theorem: Suppose f is analytic on a region and suppose $f(z) = 0$ for z in some set that has an accumulation point. Then f is identically zero on the region. (Can also be stated for power series.)

Mean Value Theorem for Complex Integrals: Suppose that $f(z)$ is analytic on $D(a;R)$ and that $0 < r < R$. Then $f(a) = \frac{1}{2\pi}\int_0^{2\pi} f(a + re^{i\theta})\,d\theta$.

Maximum Modulus Principle: For a non-constant anlytic function f on an open set D, the modulus $|f|$ cannot have a relative maximum in D. Equivalently, if $f(z)$ is analytic on a closed bounded set $\overline{D}$, then the maximum of $|f|$ must occur on the boundary of D and not at an interior point.

Minimum Modulus Principle: For a non-constant anlytic function f on an open set D, the modulus $|f|$ can have a relative minimum at a only if $f(a) = 0$.