The test will have an in-class part and a take-home part. The in-class part of the test takes place on Wednesday, December 3. It can include computational questions, statements of theorems, definitions, longer essay-type questions about concepts, and perhaps some simple proofs. The test covers Chapters 7 through 11 in the textbook, plus sections 13.1 and 16.1. However, we did not cover everything in the book.

The take-home part is scheduled to be distributed on December 3 and collected on Monday, December 7. The take-home part will consist mostly of more complex computational questions and proofs. But there might also be a general essay question. Although it will concentrate on the second half of the course, it will also, of course, require the use of definitions, theorems, and techniques from earlier in the course.

Terms and ideas that you should know:

simply connected domain (includes the interior of every simple closed curve in the domain)

multiply connected domain (connected but not simply connected)

winding number of a curve about a point, $\dfrac{1}{2\pi i}\displaystyle\int_C \frac{1}{z-a}\,dz$

deleted neighborhood of a point ($D(a;r) \setminus \{a\}$)

isolated singularity

removable singularity, pole, essential singularity

pole of order n

zero of order n

the curve $f(C)$, if C is a smooth curve and f is analytic on C

Laurent series

convergence of a Laurent series

residue of an analytic function f at a point a

why $\pi\cot(\pi z)$ and $\pi\csc(\pi z)$ can be used to computer certain infinite sums.

meromorphic function (analytic except for poles)

harmonic function

Laplace's equation

harmonic conjugate of a harmonic function

conformal function (at a point)

conformal mapping (1-to-1 analytic function)

conformal equivalence; conformally equivalent domains

linear fractional transformation, $w = \dfrac{az+b}{cz+d}$ where $ad - bc \neq 0$

A few techniques that might be on the in-class test:

Computing the residue of f at a:

- Find the Laurent series and look at the coefficient of $1/(z-a)$
- If $f(z) = \dfrac{A(z)}{z-a}$ where $A(a) \neq 0$, then $Res(f;a) = A(a)$

- If $f(z) = \dfrac{A(z)}{(z-a)^n}$ where $A(a) \neq 0$, then $Res(f;a) = \dfrac{A^{n-1}(a)}{(n-1)!}$
- If $f(z) = \dfrac{A(z)}{B(z)}$ where $A(a) \neq 0$, $B(a) = 0$, $B'(a) \neq 0$ then $Res(f;a) = \dfrac{A(a)}{B'(a)}$

Computing real integrals using residues:

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\,dx = \sum_{k=1}^{n} Res\left(\frac{P(z)}{Q(z)}; z_k\right).\ (Q(x) \text{ never zero on } \mathbb{R},\ deg(Q(x)) \geq deg(P(x)+2,$$

z_k are zeros of Q in upper half plane)

Some Important Theorems:

Open Mapping Theorem. If D is an open set and f is analytic on D, the the image, $f(D)$, is also an open set.

Morera's Theorem. If f is a continuous function on an open set D and if $\displaystyle\int_\Gamma f(z)\,dz = 0$ whenever Γ is the boundary of a rectangle in D, then f is analytic in D. (Usually stated using simple closed curves in D whose interiors are also in D.)

Cauchy Closed Curve Theorem. Suppose D is a simply connected domain, f is analytic on D, and C is a simple closed curve in D. Then $\displaystyle\int_C f(z)\,dz = 0$.

Casorati-Weierstrass Theorem. If f has an essential singularity at z_o, and if D is any deleted neighborhood of z, the $f(D)$ is dense in $\mathbb{C}$.

Laurent Series of an Analytic Function on an Annulus. Suppose that f is an analytic function on the annulus $R_1 < |z-a| < R_2$. Then f has a Laurent expansion, $\displaystyle\sum_{-\infty}^{\infty} c_k z^k$, that converges to f on the annulus.

Cauchy Residue Theorem. Suppose that f is analytic on and inside a simple closed curve C, except at isolated singularities inside C. Then $\int_C f(z)\,dz = \sum_{k=1}^{n} Res(f(z); z_k)$, where the sum is taken over all points z_k inside C where f has a singularity.

Argument Principle. Suppose that f is analytic on and inside a simple closed curve C, and that f is never zero on C. Then the number of zeros of f inside C, counting multiplicity, is $\dfrac{1}{2\pi i}\displaystyle\int_C \frac{f'(z)}{f(z)}\,dz$.

Analytic Functions are Conformal. If f is analytic at z_o and $f'(z_o) \neq 0$, then f is conformal and locally 1-to-1 at z_o.

Conformal Mapping. If f is a one-to-one analytic function from a domain D_1 onto a domain D_2, then f' is non-zero on D_1 and has an analytic inverse. Both f and f^{-1} are conformal.

Existence of Harmonic Conjugate. If $u(x,y)$ is a harmonic function on a simply connected domain D, then u is the real part of an analytic function on D.