Collected Homework Week 3

MATH 278: Number Theory
Due January 31, 2011 at 4:00pm

Name (Print): _____

1. (This was an exercise on the first groupwork sheet.) As we have discussed, number theory has a lot to do with playing with numbers and looking for patterns. Find the sum of the first two odd natural numbers. Find the sum of the first three odd natural numbers. Continue for several more sums and compare your answers. Can you find a pattern? Propose a conjecture and then prove it.

2. Repeat exercise 1 using even natural numbers. (Note: this pattern is a little harder to see, but it is there! Try finding different ways of rewriting the answers to each sum.)

3. Choose a number x such that $x > 20$.

   (a) Choose another number y such that $y > 50$ and $y \equiv x \pmod 6$. Justify why your choice of y works.

   (b) Choose another number z such that $z > 50$ and $z \not\equiv x \pmod 6$. Jusitfy why your choice of z works.

4. Prove or disprove: If $a|(b + c)$, then either $a|b$ or $a|c$.

5. Prove: $a|b$ if and only if $ac|bc$, where $c \neq 0$.