

## Collected Homework Week 4

MATH 278: Number Theory  
Due February 7, 2011 at 4:00pm

Name (Print): \_\_\_\_\_

1. Write a proof of Theorem 1.32 from page 18 of our text.
2. For each of the following, use the Euclidean Algorithm to find the GCD and then reverse the steps to express the GCD as a linear combination of  $a$  and  $b$ .
  - (a)  $a = 889$  and  $b = 229$
  - (b)  $a = 357$  and  $b = 629$
3. (a) Suppose we are interested in multiples of four. Given any positive integer, what does the Division Algorithm tell us about its possible form given this interest?
  - (b) Prove: Any perfect square will have a remainder of 1 or 0 when divided by 4.
  - (c) Given (b), what are the possible forms of a perfect square?
  - (d) Use the information you have found in the previous parts of this question to prove that  $x^2 + 6x = 450$  has no integer solution.
  - (e) Experiment with what happens when you divide perfect squares by five. Make a conjecture about divisibility by 5 similar to the one in part (b) about divisibility by 4.
  - (f) Prove your conjecture from (e).