Collected Homework Week 5

MATH 278: Number Theory
Due February 14, 2011 at 4:00pm
(Happy Valentine's Day!!!)

Name (Print): _____

1. Write a proof of Theorem 1.39 from page 19 of our text.

2. Write a proof of Theorem 1.42 from page 19 of our text.

3. Hopefully, you have been spending some quality time playing with the Questions and not being too tempted to read ahead beforehand. This is an important part of the process!!! Working through these will help make the results seem more grounded in reality and therefore stick in your brain better! To emphasize this, I would like you to share your work for Question 1.46 from page 20 of our text. If you skipped it, jump back and pretend you don't know what follows. In your "journal" you should have some scratchwork. If not, create it now. I would like you to turn in a commentary with your work that describes it. Thus you will share your calculations that you did to experiment, but also have descriptions about why you chose to proceed with those calculations, how one calculation influenced your next calculation, and what conclusions you made from them. This doesn't need to be incredibly long, but you should have a few short paragraphs of complete sentences that decribe the process.

4. **More Fun with Perfect Squares**: In the last homework you should have proved both parts of the following theorem:

Theorem: If N is a perfect square then:
(i) when N is divided by 4 the remainder is either 0 or 1.
(ii) when N is divided by 5 the remainder is either 0, 1 or 4.

(a) Prove that no perfect square can end with the digit 3 or the digit 7.

(b) Prove that no integer in the sequence 11, 111, 1111, 11111, 111111,...is a perfect square.

Notebook Problems Week 5

Fun with Greatest Common Divisors

(1) Prove that if $(a, b) = d$, then $(a/d, b/d) = 1$.

(2) Prove that if $(a, b) = 1$ and $c|(a + b)$, then $(a, c) = (b, c) = 1$.

(3) Prove that if $a|bc$, then $a|(a, b)(a, c)$. (That is, a divides the product of those gcd's).