

## Collected Homework Week 10

MATH 278: Number Theory  
Due: March 28, 2011 at 4:00pm

Name (Print): \_\_\_\_\_

1. Prove the following statements:

(a) Show that  $53^{103} + 103^{53}$  is divisible by 39.

(b) Show that 97 divides  $2^{48} - 1$  using congruences. Then find some other prime factors of  $2^{48} - 1$ . (Feel free to use any results we have proved previously!)

(c) Prove that for all natural numbers  $n$ ,  $13|3^{n+2} + 4^{2n+1}$ .

2. (a) Develop a divisibility criteria statement similar to the statements in Corollaries 9 and 10 about divisibility by 11.

(b) A palindrome is a number which reads the same forward or backward like 17244271. Show that all palindromes with an even number of digits will be divisible by 11.

## Notebook Problem Week 10

1. Find the remainder when  $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 99^5 + 100^5$  is divided by 4 using congruences.