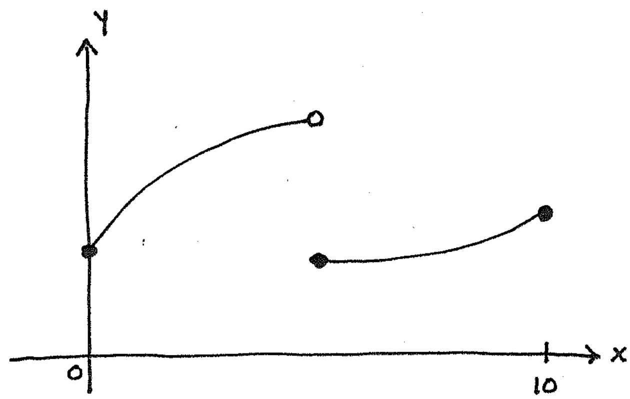
LAB Week 11

MATH 130 Section 2
April 11, 2019
Covering Sections 4.1 and 4.2

Your Name (Print): ANSWER KEY

1. Draw the graph of a function on $[0, 10]$ which has no absolute max or explain why this is impossible.

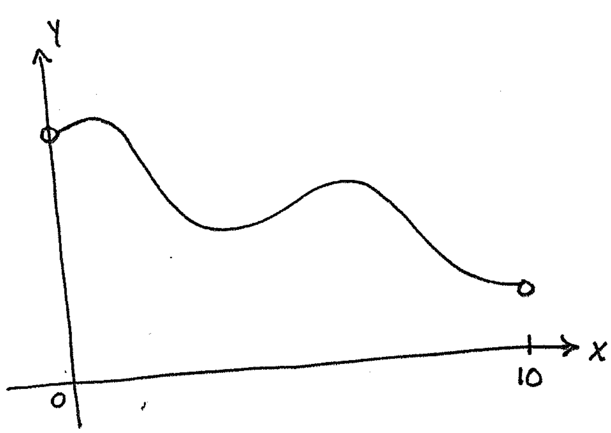


* Note there are many graphs that will work.

2. Draw the graph of a function continuous on $[0, 10]$ which has no absolute min or explain why this is impossible.

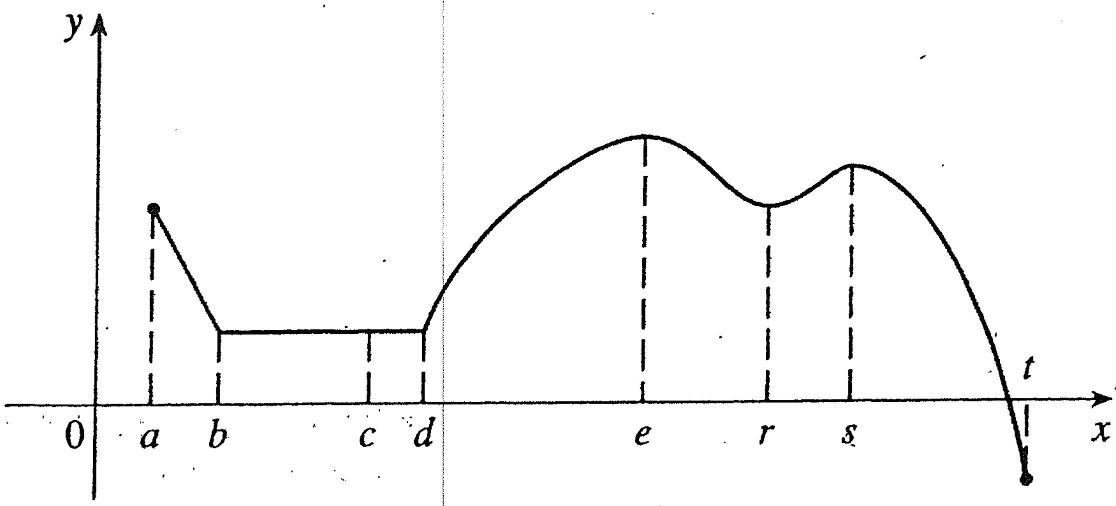
If a function is continuous on a closed interval, then it must have an absolute minimum (and an absolute maximum) on the interval by the Extreme Value Theorem. Thus this is impossible.

3. Draw the graph of a continuous function on $(0, 10)$ which has no absolute min or explain why this is impossible.



* Note there are many graphs that will work.

4. Given the graph of $y = f(x)$ below, state whether at each of the numbers a, b, c, d, e, r, s, and t the function f has an absolute maximum or minimum, a local minimum or maximum, or neither a maximum nor a minimum.



At $x=a$: neither a maximum nor a minimum

$x=b$: a local minimum

$x=c$: both a local maximum and a local minimum

$x=d$: a local minimum

$x=e$: both a local and an absolute maximum

$x=r$: a local minimum

$x=s$: a local maximum

$x=t$: an absolute minimum

5. Determine where the absolute extrema of $y = x^x$ on the interval $[.2, 1]$ occur. Be sure to show all your work and make clear that you have checked all possibilities (for example, there are two possibilities where you can have critical points; show that you have checked both).

Find the critical points:

$$y = x^x$$

$$\ln|y| = \ln|x^x|$$

$$= x\ln|x|$$

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \ln|x| \Rightarrow \frac{dy}{dx} = y\left[\frac{x}{x} + \ln|x|\right]$$

$$\frac{dy}{dx} = x^x\left[1 + \ln|x|\right]$$

$\frac{dy}{dx} = 0$ when $1 + \ln|x| = 0 \Rightarrow \ln|x| = -1 \Rightarrow x = e^{-1}$ and $\frac{1}{e}$ is in $[.2, 1]$

$\frac{dy}{dx}$ exists for all x in $[.2, 1]$

Thus the function has one critical point at $x = \frac{1}{e}$.

Test values:

$$f(.2) = .2^{.2} \approx .725$$

$$f\left(\tfrac{1}{e}\right) = \left(\tfrac{1}{e}\right)^{\frac{1}{e}} \approx .692$$

$$f(1) = 1^1 = 1$$

Thus an absolute minimum occurs at $x = \frac{1}{e}$ and an absolute maximum occurs at $x = 1$.

6 Verify that $f(x) = \frac{1}{x^2} - \frac{4}{3x} + \frac{1}{3}$ satisfies the three hypotheses of Rolle's Theorem on the interval [1,3]. (You should have complete sentences which thoroughly explain why the hypotheses hold. Note that one way to verify the differentiability requirement is to find the derivative and show that it exists on the needed interval.)

Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

(1) $f(x) = \frac{3-4x+x^2}{3x^2}$, So f is a rational function with domain $(-\infty,0) \cup (0,\infty)$. Since rational functions are continuous on their domains, f is certainly continuous on $[1,3]$.

(2) $$f'(x) = \frac{(3x^2)(-4+2x) - (3-4x+x^2)(6x)}{(3x^2)^2}$$

$$= \frac{\cancel{3}x\left[x(-4+2x) - 2(3-4x+x^2)\right]}{3\cancel{9}x^{\cancel{4}3}}$$

$$= \frac{-4x+\cancel{2x^2}-6+8x-\cancel{2x^2}}{3x^3} = \frac{-6+4x}{3x^3} = \frac{2(2x-3)}{3x^3}$$

Since $f'(x)$ is defined everywhere except $x=0$, f is differentiable on $(1,3)$.

(3) $f(1) = \frac{3-4+1}{3} = 0$

$f(3) = \frac{3-12+9}{27} = \frac{0}{27} = 0$

Since $f(1) = 0 = f(3)$, $f(a) = f(b)$.

Thus by Rolle's Theorem, there exists a c in $(1,3)$ such that $f'(c) = 0$.

$$f'(c) = \frac{2(2c-3)}{3c^3} = 0$$

$$\Rightarrow 2c-3 = 0 \Rightarrow c = \frac{3}{2}.$$

ANSWERS not SOLUTIONS

7. Let $f(x) = x^3 + ax^2 + bx$ where a and b are constants. Given that f has critical numbers at $x = 1$ and at $x = 3$, find a and b.

$a = -6$

$b = 9$

8. For each of the following statements, state whether it is true or false. If the statement is false, give a counterexample. If the statement is true, explain why/how you know.

(a) If $f'(c) = 0$, then $f(x)$ has a maximum or minimum value at $x = c$.

FALSE ... why?

(b) If $f'(x) = g'(x)$ for all x in an interval I, then $f(x) = g(x)$ on I.

FALSE ... why?

(c) If $f(x)$ is differentiable on the open interval (a, b), and c is a point of local maximum for f in (a, b), then $f'(c) = 0$.

TRUE ... why?

9. Find the critical numbers, if any, of the following functions. Carefully explain how you have checked all possibilities (i.e. there are two and you should make it clear that you have checked both).

(a) $f(x) = x - \arctan x$

critical number : $x = 0$

(b) $g(\theta) = 2\sec\theta + \tan\theta$ for $0 < \theta < 2\pi$

critical numbers : $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$