LAB Week 14

MATH 130 Section 2 Your Name (Print): ANSWER KEY

May 2, 2019 Covering Sections 4.4, 4.9 and a review of some topics from earlier in the semester.

Bring this lab to class with you on Monday. We will spend additional time working on it then. This is a list of problems and it is not expected that enough room is provided for you to do all of them here. It is not necessary to do them in order, choose on which section you would like to focus.

Part 1: Optimization

1. Find two numbers whose difference is 100 and whose product is a minimum.

$x = 50$, $y = -50$

2. The management of the Lowes store has decided to enclose an 800-square-foot area outside the building for displaying evergreen trees and wreaths. One side will be formed by the external wall of the store, the side parallel to the wall of the store will be made of galvanized steel fencing material, and the other two sides will be constructed of pine boards. If the pine board fencing costs \$6 per running foot and the steel fencing costs \$3 per running foot, determine the dimensions of the enclosure that can be erected at minimum cost.

x (length of side of pine) $= 10\sqrt{2}$ ft, $y = 40\sqrt{2}$ ft

3. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used by the square and how much should be used for the circle to enclose the maximum total area? Intuitively, you may guess the answer, but be sure you can prove it using the same methods you used on the problems above.

All the wire should be used for the circle! (extremum at endpoint!)

Part 2: True/False and Multiple Choice Questions

The final exam will have both True/False and Multiple Choice questions on it. When answering the True/False questions below, be sure to understand why your answer is correct and think about how you might reword the statement to have a different answer. Most of the Multiple Choice questions are regular problems. Work out your solution and find it among the choices given; try not to look at the answers first. Note that no particular work is required for the Multiple Choice questions. Only the circled answer is considered.

4. **True/False**

Circle T if the statement is **always** true or F if the statement is not always true.

T F If $f(x)$ has a local maximum at $x = a$, then $f'(a) = 0$.

T F The inverse function of $y = \log_8 x$ is $y = 8^x$.

T F If a function $f(x)$ is differentiable at a, then $f(x)$ is continuous at a.

T F If $f(x) = \dfrac{x^2 - 25}{x - 5}$ and $g(x) = x + 5$, then $f(x) = g(x)$.

5. **Multiple Choice**

Circle the number which best answers the question or completes the statement.

(A) Suppose that $\lim_{x\to -3}\left(4-\frac{5x}{3}\right)=9$. Let $\epsilon>0$. Which of the following is an appropriate value for δ in the proof of the limit statement?

(i) 0

(ii) $-\frac{3}{5}\epsilon$

(iii) $\frac{3}{5}\epsilon$

(iv) $\frac{5}{3}\epsilon$

(v) none of the above

(B) $\lim_{x\to 3^-}\frac{2-x}{x(x-3)}=$

(i) ∞

(ii) $-\infty$

(iii) 0

(iv) -1

(v) 1

Part 3: Revisiting some old friends

6. Evaluate the following limits if they exist. If the limit does not exist, explain why.

(a) $\lim_{x\to 7^+}\frac{x-1}{7-x}$ $= -\infty$ (DNE)

(b) $\lim_{x\to 5}\frac{x^2+2x-35}{|x-5|}$ DNE since the right hand limit does not equal the left hand limit

(c) $\lim_{x\to \frac{\pi}{2}^-}(\sin x)^{\tan x}$ $= 1$

7. Find the vertical asymptotes of $f(x) = \dfrac{3x^2 - 7x - 6}{2x^2 - 18}$. Explicitly state the equations of any vertical asymptotes you find, justify why each is one, and why you know you have found them all.

$x = -3$ is the only vertical asymptote ... why?

8. Let $f(x) = \dfrac{(x+1)^2}{x^2 - 1}$.

(a) At what value(s), if any, does f have a removable discontinuity? Justify your answer.

$x = -1$ (solutions should include limits!)

(b) At what value(s), if any, does f have an infinite discontinuity? Justify your answer.

$x = 1$ (solutions should include limits!)

9. Find the equation of a function that has a jump discontinuity at $x = 2$. Justify why your example works.

One example: $f(x) = \dfrac{|x-2|}{x^2 - x - 2}$ → can you explain why?

10. Differentiate each of the following functions and simplify your answers.

(a) $f(x) = 37 - x^{\sqrt{2}} + \sec^3(2x) + \sin x \log_3 x$

$$f'(x) = -\sqrt{2}\, x^{\sqrt{2}-1} + 6\sec^3(2x)\tan(2x) + \frac{\sin x}{x\ln 3} + \cos x \log_3 x$$

(b) $y = 5^{x^2} + \ln(\operatorname{arcsec}(x^5))$

$$\frac{dy}{dx} = 5^{x^2}(2x\ln 5) + \frac{5x^4}{|x^5|\operatorname{arcsec}(x^5)\sqrt{x^{10}-1}}$$

11. Find $\dfrac{d^2y}{dx^2}$ if $x^4 + y^4 = 16$.

$$\frac{d^2y}{dx^2} = \frac{-3x^2y^4 - 3x^6}{y^7}$$

12. Find $D^{50} \cos 5x$. [That is, can you find the 50th derivative of $\cos 5x$ without taking 50 derivatives?]

$$D^{50} \cos 5x = -5^{50} \cos(5x)$$

13. Find an equation of the tangent line to $f(x) = \dfrac{5}{1+x^2}$ at the point (-2,1).

$$y - 1 = \frac{4}{5}(x+2)$$

14. Two straight roads intersect at right angles in Geneva. Car A is on one road moving toward the intersection at a speed of 50 miles/hr. Car B is on the other road moving away from the intersection at a speed of 30 miles/hr. When car A is 2 miles from the intersection and car B is 4 miles from the intersection, how fast is the distance between the cars changing?

$\sqrt{20}$ m/hr

15. Find the absolute extrema of $f(x) = 3x^{\frac{2}{3}} - 2x$ on [-1,1]. Give complete coordinates.

Absolute maximum: $(-1, 5)$

Absolute minimum: $(0, 0)$

Part 4: Antiderivatives

Try these questions after class on Friday!

16. Evaluate the following integrals. Don't forget your family! The first two require some manipulation before beginning. For the rest, I would like you to think about patterns and about what the function might look like whose derivative is the given integrand (remember, the integrand is the function you are trying to find the antiderivative of).

(a) $\displaystyle\int x^4(x^5 + x^2 + 7)dx = \frac{x^{10}}{10} + \frac{x^7}{7} + \frac{7x^5}{5} + C$

(b) $\displaystyle\int \frac{x^3 \cos x + x \sec x + \cos x}{x \cos x} dx = \frac{x^3}{3} + \tan x + \ln|x| + C$

(c) $\displaystyle\int 4 \sin^3 \theta \cos \theta d\theta = \sin^4 \theta + C$

(d) $\displaystyle\int 2x\sqrt{x^2 - 9}dx = \frac{2}{3}(x^2-9)^{3/2} + C$

(e) $\displaystyle\int 7x^6 \cos(x^7 + 23)dx = \sin(x^7 + 23) + C$