LAB Week 2

MATH 130 Section 2
January 31, 2019
Covering Precalculus

Your Name (Print): ANSWER KEY

Simplify your answers so that you have gathered all like terms, cancelled where possible, and so that there are no negative exponents or fractions within fractions in your final answer. Neatness is a plus!

1. Consider the two functions: $f(x) = 1 - x$ and $g(x) = \dfrac{1}{x}$.

Find the following new functions and specify their domains (Hint: think carefully about how we find domains of compositions of functions!).

(a) $g(f(x)) = g(1-x)$

$$= \frac{1}{1-x}$$

Domain: $\mathbb{R} - \{1\} = (-\infty, 1) \cup (1, \infty)$

(b) $g(g(x)) = g\left(\frac{1}{x}\right)$

$$= \frac{1}{\frac{1}{x}}$$

$$= x$$

Domain: $\mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$

*Remember that the domain of $f(g(x))$ will always be a subset of the domain of $g(x)$.

2. Look carefully at your results in part (b) of the previous question. What does this tell you about the function g in question 1? Explain your conclusion in a full sentence.

Since $g(g(x)) = x$ (and composing in either order is the same), g is its <u>own</u> inverse!

3. Evaluate the following. Justify your answers with work and/or a full sentence.

(a) $\arctan\left(\tan\frac{11\pi}{6}\right) = \arctan\left(-\frac{1}{\sqrt{3}}\right)$

$= -\frac{\pi}{6}$

Note: $\frac{11\pi}{6}$ is not in the restricted domain for

$\tan x$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

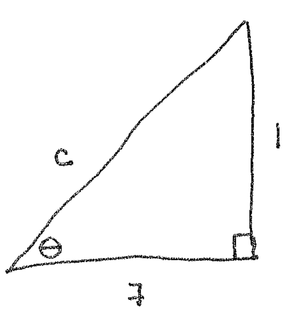
(b) $\arccos\left(\cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$

Since $\arccos x$ and $\cos x$ are inverse functions and $\frac{5\pi}{6}$ is in the restricted domain of $\cos x$: $[0, \pi]$, the output is the same as the input.

(c) $\sin\left(\arcsin\frac{7}{5}\right)$ is undefined since $\frac{7}{5}$ is not in the domain of $\arcsin x$: $[-1, 1]$.

(d) $\sin(\text{arccot } 7)$

We want to find $\sin\theta$ where $\theta = \text{arccot } 7$ or $\cot\theta = \frac{7}{1}$.



So $c^2 = 1^2 + 7^2 = 1 + 49 = 50$

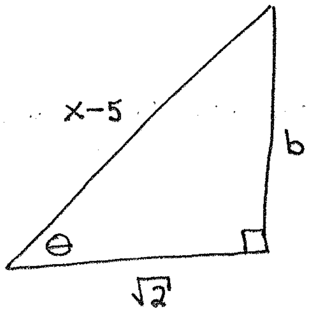
Thus $c = \sqrt{50} = 5\sqrt{2}$

Hence $\sin\theta = \frac{1}{5\sqrt{2}}$.

(e) $\tan\left(\operatorname{arcsec}\dfrac{x-5}{\sqrt{2}}\right)$

We want to find $\tan\theta$ where $\theta = \operatorname{arcsec}\dfrac{x-5}{\sqrt{2}}$

or $\sec\theta = \dfrac{x-5}{\sqrt{2}}$



So $(x-5)^2 = b^2 + (\sqrt{2})^2$

$b^2 = (x-5)^2 + 2 = x^2 - 10x + 23 - 2 = x^2 - 10x + 23$

$b = \sqrt{x^2 - 10x + 23}$

Thus $\tan\theta = \dfrac{\sqrt{x^2-10x+23}}{\sqrt{2}}$

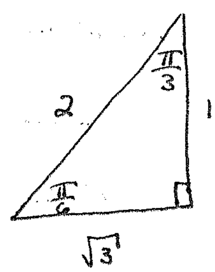
4. Solve the following: $2\theta\cos\theta + \theta = 0$ where $0 \le \theta < 2\pi$.

$\theta(2\cos\theta + 1) = 0$

Thus $\theta = 0$ or $2\cos\theta + 1 = 0$

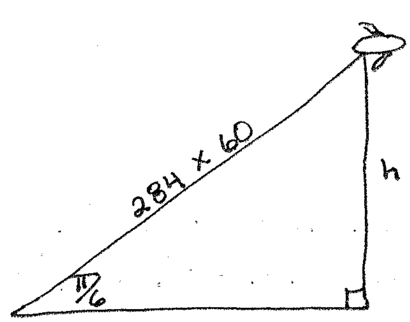
$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$



Therefore possible solutions are: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

5. An airplane leaves the runway climbing at an angle of $\frac{\pi}{6}$ radians with a speed of 284 feet per second. Draw a diagram (hint: use a triangle) representing the flight path of the airplane in relationship to the ground. Find the altitude of the plane after 1 minute.



After 60 seconds the plane will have traveled $284 \times 60 = 17{,}040$ ft.

$\sin\left(\frac{\pi}{6}\right) = \dfrac{h}{17{,}040}$

$h = \sin\left(\frac{\pi}{6}\right) * 17{,}040 = \frac{1}{2} * 17040$

$= 8520$ ft

6. Review the properties of exponents (on your green prerequisites sheet) and logarithms (in the sidebar on page 33 in your text). Then solve the following for x by exponentiating or taking the logarithm of both sides. (In general, your answers to these types of questions should be exact and therefore may contain e or logs.)

(a) $e^{x^2-5x+5} = \frac{1}{e}$

$$e^{x^2-5x+5} = e^{-1}$$

$$\ln\left(e^{x^2-5x+5}\right) = \ln(e^{-1})$$

$$x^2-5x+5 = -1$$

$$x^2-5x+6 = 0$$

$$(x-3)(x-2) = 0$$

So $x=3$ and $x=2$ are both solutions.

(b) $\ln(3x+1) + \ln x = \ln 2$

$$\ln[(3x+1)x] = \ln 2$$

$$e^{\ln(3x^2+x)} = e^{\ln 2}$$

$$3x^2+x = 2$$

$$3x^2+x-2 = 0$$

$$(3x+2)(x-1) = 0$$

$3x+2=0$ $\qquad$ $x-1=0$

$x=-\frac{2}{3}$ $\qquad$ $x=1$

7. Find the domain of the following functions. Justify your answers. Give your solution in interval notation.

(a) $f(x) = \ln\left(\dfrac{x+6}{x-2}\right)$

The domain of $\ln x$ is $(0,\infty)$, thus we need $\frac{x+6}{x-2} > 0$.

$\frac{x+6}{x-2}$ $\quad$ + + + 0 − − − U + + +

$\qquad\qquad$ −6 $\qquad\qquad$ 2

$x+6$ $\quad$ − − − 0 + + + + +

$x-2$ $\quad$ − − − − − − − 0 + +

Thus the domain of $\ln\left(\frac{x+6}{x-2}\right)$ is $(-\infty,-6) \cup (2,\infty)$.

(b) $g(x) = e^{\sqrt{5-x}}$

The function e^x is defined for all real numbers so the only restriction is for $\sqrt{5-x}$ and we need $5-x \geq 0$. Thus the domain is $(-\infty, 5]$

* These are answers not solutions ! *

You will probably need additional paper to complete the remaining problems.

8. Given f and g defined by the following table:

x	-3	-2	-1	0	1	2	3
f(x)	1	0	1	2	3	4	5
g(x)	7	2	-1	-2	-1	2	7

Determine the following (remember to show your work!):

(a) $(f-g)(2) = 2$

(b) $(f/g)(-3) = \frac{1}{7}$

(c) $(f \circ g)(1) = 1$

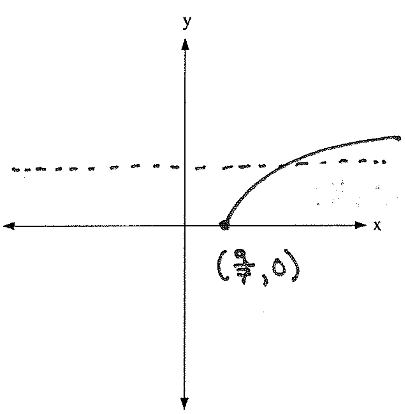
(d) $(f \circ f)(0) = 4$

(e) $(g \circ f)(1) = 7$

(f) Review your answer in part (c). Does this mean that f and g are inverse functions? Why or why not? No! Why?

9. In this question we will refresh our memories with an outline of the steps needed to find the inverse of a function, and apply those steps to the function $f(x) = \sqrt{7x-9}$. (Note that if you learned this previously with steps 2 and 3 interchanged, you may do it that way!)

Step 1: Determine if f has an inverse. Recall that this is equivalent to showing a function is one-to-one. This can be done in two different ways, using the Horizontal Line Test or using the definition of one-to-one. Use the Horizontal Line Test here to show that f has an inverse.



Step 2: Solve $y = f(x)$ for x as a function of y. $x = \frac{y^2+9}{7}$

Step 3: Express f^{-1} as a function of x by interchanging x and y in your result from Step 2.

$f^{-1}(x) = (x^2+9)/7$

Step 4: Specify the DOMAIN of f^{-1}!!!

$[0, \infty)$

Step 5: Check your result by showing $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. (It is wise to always do this, and you should do it now, but you need not show me this step in general.)

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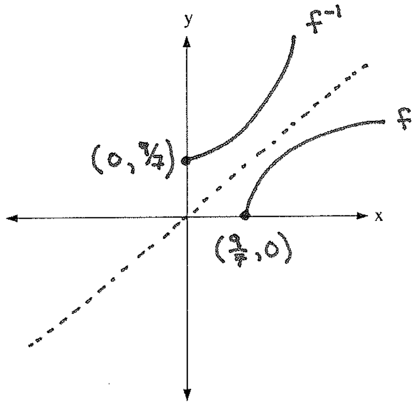
10. Graph the function f from number 9 again. Then, using your knowledge of **graphs** of inverse functions and without using the equation you found in the previous question, graph f^{-1}. Briefly explain your process.



11. (a) The exponential function we see the most, because it is found in nature and so forth, is $f(x) = e^x$ and is called the natural exponential function. Remember that e is a number! Write down the first thirteen digits of e.

$e \approx 2.718281828459$

(b) What are the domain and range of $f(x) = e^x$? What is the inverse function of $f(x) = e^x$? What are the inverse's domain and range? (Be sure that it is clear which question you are answering.)

D: $(-\infty, \infty)$; R: $(0, \infty)$; $y = \ln x$ with D: $(0, \infty)$ & R: $(-\infty, \infty)$

(c) Fill in the blank: $y = e^x$ if and only if $\ln y = x$. (Hint: this has to do with the inverse of $y = e^x$.)

12. Find the coordinates of a second point on the graph of a function f if $(7, -3)$ is on the graph and the function is

(a) odd $(-7, 3)$

(b) even $(-7, -3)$

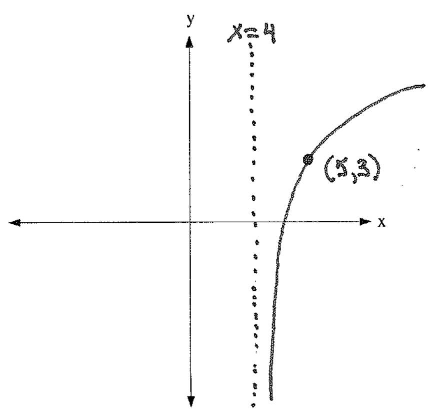
13. Use the properties of logarithms to express $\frac{1}{3}[2\ln(x+3) + \ln \tan x - \ln(x^2 - 1)]$ as the logarithm of a single quantity.

$\ln\left[\frac{(x+3)^2 \tan x}{x^2 - 1}\right]^{1/3}$

14. Assume that f is one-to-one (i.e. you can skip Step 1). Find the inverse of f if $f(x) = \dfrac{e^x}{1 + 2e^x}$. Remember each step.

$f^{-1}(x) = \ln\left(\frac{x}{1 - 2x}\right)$ with domain $(0, \frac{1}{2})$

15. Sketch the graph of the function $y = \ln(x - 4) + 3$. Be sure to label important features.



$$f(x) = \begin{cases} 2x+3 & \text{if } x \geq 0 \\ 3 & \text{if } -3 \leq x < 0 \\ -2x-3 & \text{if } x < -3 \end{cases}$$

16. Express $f(x) = |x| + |x + 3|$ without using absolute value signs. Justify your answer.