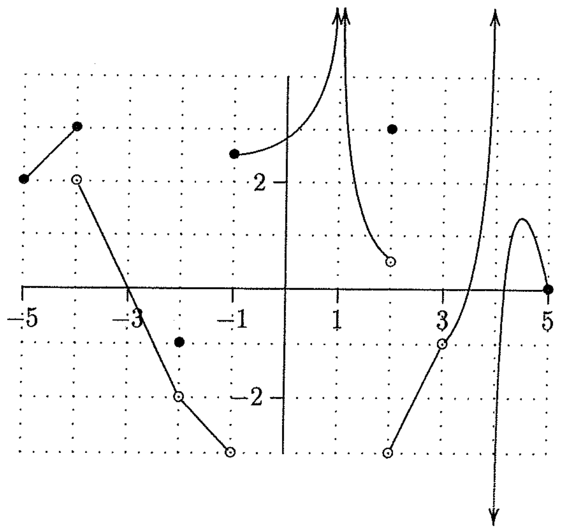
LAB Week 3

MATH 130 Section 2
February 7, 2018
Covering Sections 2.1 and 2.2

Your Name (Print): ANSWER KEY

1. Fill in the table. Use $+\infty$ and $-\infty$ where appropriate.



a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$
-4	3	2	DNE	3
-3	0	0	0	0
-2	-2	-2	-2	-1
-1	-3	2.5	DNE	2.5
0	2.8	2.8	2.8	2.8
1	(DNE) ∞	(DNE) ∞	(DNE) ∞	UND
2	0.5	-3	DNE	3
3	-1	-1	-1	UND
4	∞	$-\infty$	DNE	UND

2. (a) Is it possible for $\lim_{x\to a} f(x)$ to exist and $f(a)$ to be defined, but for the two values to be different? Explain your conclusion in a full sentence(s).

Yes. When evaluating $\lim_{x\to a} f(x)$ we are asking what happens to $f(x)$ as x gets close to a, but we do not care what happens at $x=a$. For example, above we have $\lim_{x\to -2} f(x) = -2$ and $f(-2) = -1$, and so $\lim_{x\to -2} f(x) \neq f(-2)$.

(b) Is it possible to have a function where $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a^+} f(x)$ exists, but for the two values to be different? Explain your conclusion in a full sentence(s).

No. By Theorem 2.1, if $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a^+} f(x) = L$ as well.

3. The point $P(3,1)$ lies on the curve $y = \sqrt{x-2}$.

(a) If Q is the point $(x, \sqrt{x-2})$, write down the formula for the slope of the secant line PQ.

$$m_{sec} = m_{PQ} = \frac{\sqrt{x-2} - 1}{x-3}$$

(b) Now use a calculator to find the slope of the secant line PQ (correct to six decimal places - do not round) for the following values of x. Place the values you find in the table. Try writing sideways.

x	2.5	2.9	2.99	2.999	3.001	3.01	3.1	3.5
m_{sec}	.585786	.513167	.501256	.500125	.499875	.498756	.488088	.449489

(c) Using the results of part (b), guess the value of the slope of the tangent line to the curve at $P(3,1)$.

$$m_{tan} = \frac{1}{2}$$

(d) Using the slope from part (c), find an equation of the tangent line to the curve at $P(3,1)$.

$$y - 1 = \frac{1}{2}(x-3)$$

or

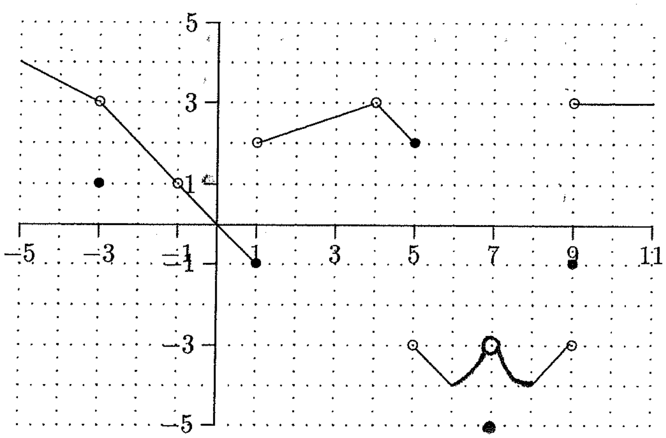
$$y = \frac{1}{2}x - \frac{3}{2} + 1 \quad \Rightarrow \quad y = \frac{1}{2}x - \frac{1}{2}$$

4. Each of the parts of this question are to be completed in the table or on the graph below.

(a) Fill in the first four columns below by using the graph to determine the limits and function values, if they exist.

(b) In Section 2.6 we will study the important concept of continuity more closely. Here we will introduce the idea in terms of reading graphs. A function f is **continuous** at a if $\lim_{x\to a} f(x) = f(a)$, i.e., if the function's value and its limit both exist and are the same number. Fill in $f(a)$ and then determine whether f is continuous (Cont) at a, responding with Y (yes) or N (no).

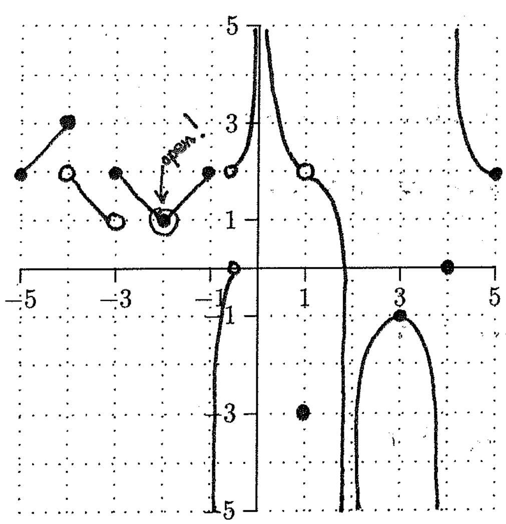
(c) Complete the graph between 6 and 8 so that $f(7) = -5$ but $\lim_{x\to 7} f(x) = -3$.



+6

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Cont
−4	3.5	3.5	3.5	3.5	Y
−3	3	3	3	1	N
−1	1	1	1	DNE	N
0	0	0	0	0	Y
1	-1	2	DNE	-1	N
4	3	3	3	DNE	N
5	2	-3	DNE	2	N
9	-3	3	DNE	-1	N

5. Fill in the table below consistent with the information given. There may be more than one correct choice for some entries. Then draw a graph which satisfies *all* of the conditions listed in the table. □ ← Means other choices work too!

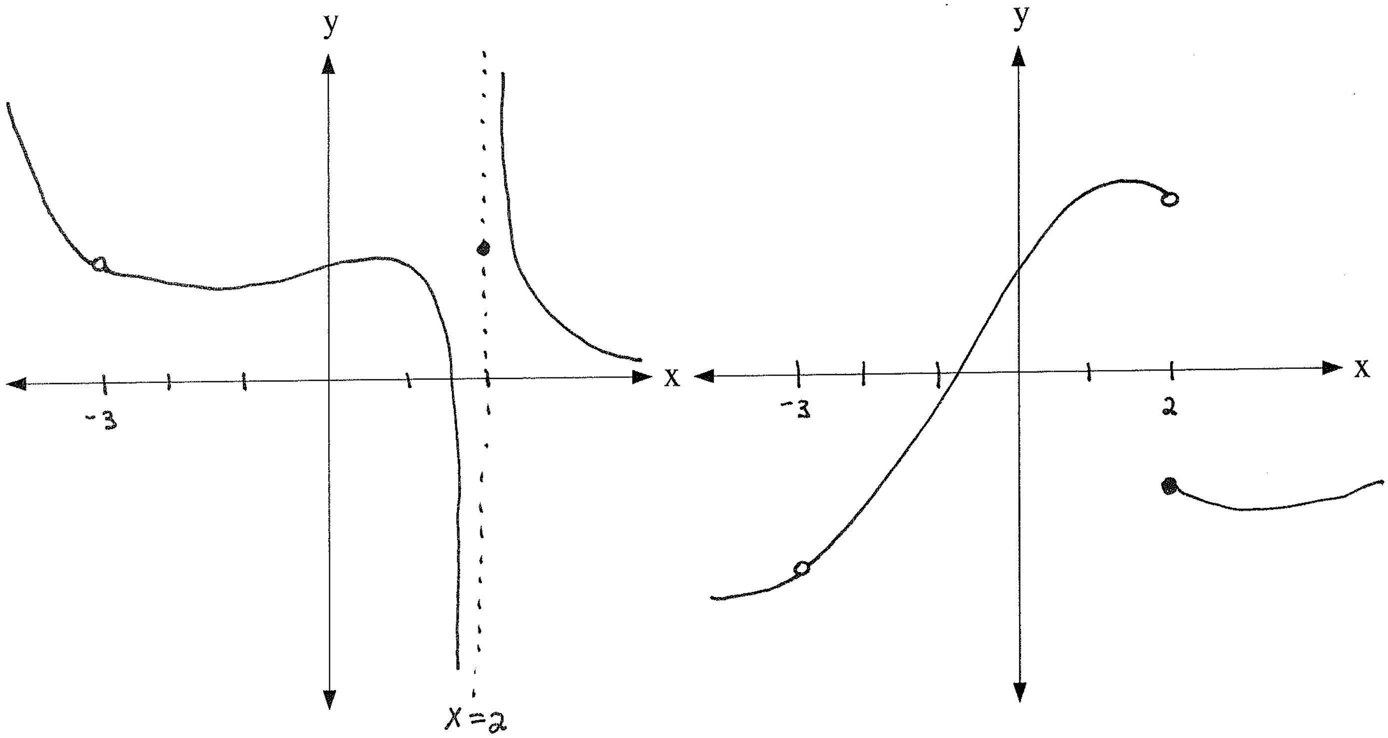


+7

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Cont
−5	—	2	—	2	—
−4	3	2	DNE	$\boxed{3}$	N
−3	1	2	DNE	2	N
−2	1	1	1	DNE	N
−1	2	$-\infty$	DNE	2	N
0	$+\infty$	$+\infty$	$+\infty$	DNE	N
1	2	2	2	$\boxed{-3}$ *	Not Cont
2	$-\infty$	$-\infty$	$-\infty$ DNE	$\boxed{\text{DNE}}$	N
3	−1	-1	-1	-1	Cont
4	$-\infty$	$+\infty$	DNE	0	N
5	2	—	—	$\boxed{2}$	Not Cont

* Not 2

6. Let $f(x)$ be a function with domain $(-\infty, -3) \cup (-3, \infty)$ whose limit does exist as x approaches -3, but does not exist as x approaches 2. Draw two different possibilities of what f might look like. Be sure both graphs fulfill all three requirements.



7. (a) Suppose g is an even function and you know that $\lim_{x\to 3^-} g(x) = 7$ and $\lim_{x\to 3+} g(x) = 4$. What other limits do you know? Explain carefully with full sentences.

Since g is even, it is symmetric about the y-axis.
We can therefore use the given limits to determine what is happening (near) (at) $x = -3$:

$$\lim_{x\to -3^-} g(x) = 4 \quad \text{and} \quad \lim_{x\to -3^+} g(x) = 7.$$

(b) Repeat part (a) assuming that g is an odd function.

Since g is odd, it is symmetric about the origin.
We can therefore use the given limits to determine what is happening near $x = -3$:

$$\lim_{x\to -3^-} g(x) = -4 \quad \text{and} \quad \lim_{x\to -3^+} g(x) = -7.$$

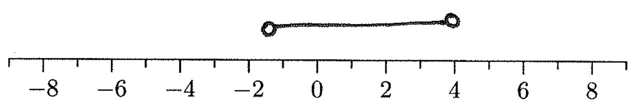
DOES NOT INCLUDE FULL SOLUTIONS

8. Solve each of the following.

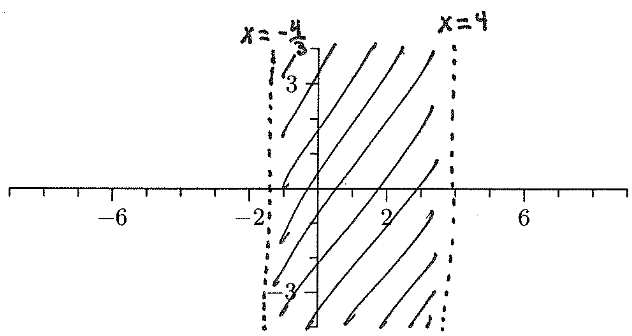
a) $|7x+4| = 16$ **b)** $|x+9| \le 13$ **c)** $|3x-4| < 8$

$x = \frac{12}{7}, \frac{-20}{7}$ $-22 \le x \le 4$ $-\frac{4}{3} < x < 4$

d) Graph the solution set to part (c) on the given number line.



e) Graph the points in the *plane* which satisfy (c).



f) Find and graph all points in the plane which satisfy both $|x+3| < 4$ and $|4y-5| < 9$.