LAB Week 6

MATH 130 Section 3
February 28, 2019
Covering Sections 2.5 and 2.6

Your Name (Print): ANSWER KEY

1. On Lab 3 you completed tables like this before we started discussing continuity in depth. Now we will add columns to evaluate whether or not the function is left continuous and right continuous at a particular value. In the last column, state whether the function is continuous at the point by typing "Yes" or "No". If you answer "No", state whether the discontinuity is "Removable", "Jump" or "Infinite". Fill in the table using the information given. For a few, several correct answers are possible.

a	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$	Left Cont?	Right Cont?	Cont? If not, kind of discontinuity?
-4	4	4	4	4	Yes	Yes	Yes
-3	1	2	DNE	3	No	No	No, Jump
-2	-3	-3	-3	$\boxed{0}$	No	No	No, Removable
-1	$\boxed{5}$	3	DNE	3	No	Yes	No, Jump
0	6	6	6	6	Yes	Yes	Yes
1	4	3	DNE	4	Yes	No	No, Jump
2	2	2	2	$\boxed{7}$	No	No	No, Removable
3	$\boxed{\infty}$	-1	DNE	-1	No	Yes	No, $\boxed{\text{Infinite}}$
4	5	-5	DNE	5	Yes	No	No, Jump
5	4	4	4	4	Yes	Yes	Yes
6	7	7	7	7	Yes	Yes	Yes
7	3	3	3	$\boxed{\text{DNE}}$	No	No	No, Removable
8	$\boxed{8}$	$\boxed{8}$	$\boxed{8}$	1	No	No	No, Removable

$\square$ means other answers possible, though not all possibilities are correct!

2. Find the horizontal asymptotes of $f(x) = \dfrac{8x^3 - 10x^2 + 13}{5x^3 - \sqrt{16x^6 + 7x^2 - 4}}$, and justify that you have found them all.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{8x^3 - 10x^2 + 13}{5x^3 - \sqrt{16x^6 + 7x^2 - 4}} \cdot \frac{\frac{1}{\sqrt{x^6}}}{\frac{1}{\sqrt{x^6}}} \qquad \sqrt{x^6} = x^3 \text{ since } x > 0$$

$$= \lim_{x\to\infty} \frac{\frac{8x^3}{x^3} - \frac{10x^2}{x^3} + \frac{13}{x^3}}{\frac{5x^3}{x^3} - \sqrt{\frac{16x^6 + 7x^2 - 4}{x^6}}} = \lim_{x\to\infty} \frac{8 - \frac{10}{x} + \frac{13}{x^3}}{5 - \sqrt{\frac{16x^6}{x^6} + \frac{7x^2}{x^6} - \frac{4}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{8 - \frac{10}{x}^{\to 0} + \frac{13}{x^3}^{\to 0}}{5 - \sqrt{16 + \frac{7}{x^4}^{\to 0} - \frac{4}{x^6}^{\to 0}}} = \frac{8}{5 - \sqrt{16}} = \frac{8}{5-4} = 8$$

so $y = 8$ is a horizontal asymptote.

$$\lim_{x\to-\infty} \frac{8x^3 - 10x^2 + 13}{5x^3 - \sqrt{16x^6 + 7x^2 - 4}} \cdot \frac{\frac{1}{\sqrt{x^6}}}{\frac{1}{\sqrt{x^6}}} \qquad \sqrt{x^6} = -x^3 \text{ since } x < 0$$

$$= \lim_{x\to-\infty} \frac{\frac{8x^3}{-x^3} - \frac{10x^2}{-x^3} + \frac{13}{-x^3}}{\frac{5x^3}{-x^3} - \sqrt{\frac{16x^6 + 7x^2 - 4}{x^6}}} = \cdots = \lim_{x\to-\infty} \frac{-8 + \frac{10}{x}^{\to 0} - \frac{13}{x^3}^{\to 0}}{-5 - \sqrt{16 + \frac{7}{x^4}^{\to 0} - \frac{4}{x^6}^{\to 0}}}$$

$$= \frac{-8}{-5 - \sqrt{16}} = \frac{-8}{-5-4} = \frac{8}{9}$$

so $y = \frac{8}{9}$ is a horizontal asymptote of f.

3. Evaluate the following limits. Use the fact that transcendental functions are continuous on their domains where appropriate. Be sure to use good limit grammar!

(a) $\lim_{x\to\infty} \dfrac{\overset{\to \pi/2}{\arctan x}}{\underset{\to \infty}{4x^3-5x^2+7}} = 0$

(b) $\lim_{x\to-\infty} \dfrac{5x^3-\sqrt{16x^6+7x^2-4}}{8x^3-10x^2+13} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty} \dfrac{\dfrac{5x^3}{x^3} - \dfrac{\sqrt{16x^6+7x^2-4}}{-\sqrt{x^6}}}{\dfrac{8x^3-10x^2+13}{x^3}}$ $\quad x^3=-\sqrt{x^6}$ since $x<0$

$= \lim_{x\to-\infty} \dfrac{5+\sqrt{\dfrac{16x^6+7x^2-4}{x^6}}}{\dfrac{8x^3}{x^3}-\dfrac{10x^2}{x^3}+\dfrac{13}{x^3}} = \lim_{x\to-\infty} \dfrac{5+\sqrt{\dfrac{16x^6}{x^6}+\dfrac{7x^2}{x^6}+\dfrac{-4}{x^6}}}{8-\dfrac{10}{x}+\dfrac{13}{x^3}}$

$= \lim_{x\to-\infty} \dfrac{5+\sqrt{16+\underbrace{\dfrac{7}{x^4}}_{\to 0}-\underbrace{\dfrac{4}{x^6}}_{\to 0}}}{8-\underbrace{\dfrac{10}{x}}_{\to 0}+\underbrace{\dfrac{13}{x^3}}_{\to 0}} = \dfrac{5+\sqrt{16}}{8} = \dfrac{5+4}{8} = \dfrac{9}{8}$

(c) $\lim_{x\to\pi/2} \dfrac{\sin x-1}{\sqrt{\sin x}-1} \cdot \dfrac{\sqrt{\sin x}+1}{\sqrt{\sin x}+1} = \lim_{x\to\pi/2} \dfrac{\cancel{(\sin x-1)}(\sqrt{\sin x}+1)}{\cancel{\sin x-1}}$

$= \lim_{x\to\pi/2} (\sqrt{\sin x}+1)$

$= \sqrt{\sin \frac{\pi}{2}}+1$ since $\sin x$ is continuous on its domain

$= 1+1$

$= 2$

4. Let $f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1} & \text{if } x > -1, \\ 1 & \text{if } x = -1,. \\ \dfrac{x^2+x+2}{x+1} & \text{otherwise.} \end{cases}$

(a) Is f left continuous at -1? Justify your answer with limits and sentences.

$$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} \frac{\overbrace{x^2+x+2}^{\to 2}}{\underbrace{x+1}_{\to 0^-}} = -\infty$$

Since $\lim_{x\to-1^-} f(x)$ does not exist, f is <u>not</u> left continuous at $x=-1$.

(b) Is f right continuous at -1? Justify your answer with limits and sentences.

$$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} \frac{x^2+3x+2}{x+1} = \lim_{x\to-1^+} \frac{\cancel{(x+1)}(x+2)}{\cancel{x+1}}$$

$$= \lim_{x\to-1^+} (x+2) = 1 \qquad \text{also } f(-1) = 1$$

Since $\lim_{x\to-1^+} f(x) = f(-1)$, f <u>is</u> right continuous at $x=-1$.

(c) Is f continuous at -1? If it is, explain. If it is not, determine what kind of discontinuity it is and justify your answer using definitions!

Since $\lim_{x\to-1^-} f(x) \neq \lim_{x\to-1^+} f(x)$, $\lim_{x\to-1} f(x)$ does not exist and so f is <u>not</u> continuous at $x=-1$.

(d) Determine where the function is continuous. Express your answer as the union of two or more intervals. Use your theorems!

Since $\dfrac{x^2+3x+2}{x+1}$ is a rational function, it is continuous everywhere $x+1 \neq 0$. Thus f is continuous on $(-1,\infty)$. Similarly, $\dfrac{x^2+x+2}{x+1}$ is continuous on $(-\infty,-1)$. Thus since we showed in (c) that f is not continuous at $x=-1$, we have f is continuous on $(-\infty,-1)\cup(-1,\infty)$.

5. If $f(x) = x^2 + 10\sin\left(\frac{\pi x}{2}\right)$, show that there is a number c such that $f(c) = 1000$. Try solving this without using a calculator (choose numbers wisely!). To solve this you should be using (and stating that you are using!) a theorem and you should not actually find the c, just show that it exists!

Since x^2 is a polynomial, it is continuous everywhere. Since $10\sin\left(\frac{\pi x}{2}\right)$ is a trigonometric function, it is continuous on its domain, which is everywhere. Thus, since the sum of continuous functions is continuous, $f(x)$ is continuous everywhere.

+4

Note: $f(10) = 10^2 + 10\sin(5\pi) = 100 + 0 = 100 < 1000$ and

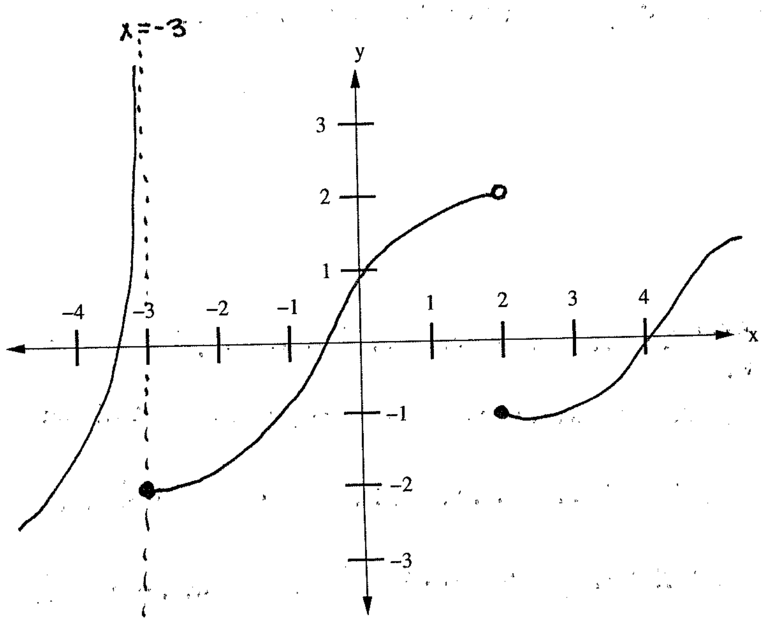
$f(40) = 40^2 + 40\sin(20\pi) = 1600 + 0 = 1600 > 1000$

Since $f(10) < 1000$ and $f(40) > 1000$, there exists a c in $(10, 40)$ such that $f(c) = 1000$ by the Intermediate Value Theorem.

∎

6. Sketch the graph of a function f that has an infinite discontinuity at $x = -3$, a jump discontinuity at $x = 2$, is continuous everywhere else, and $f(-3) = -2$.

+3

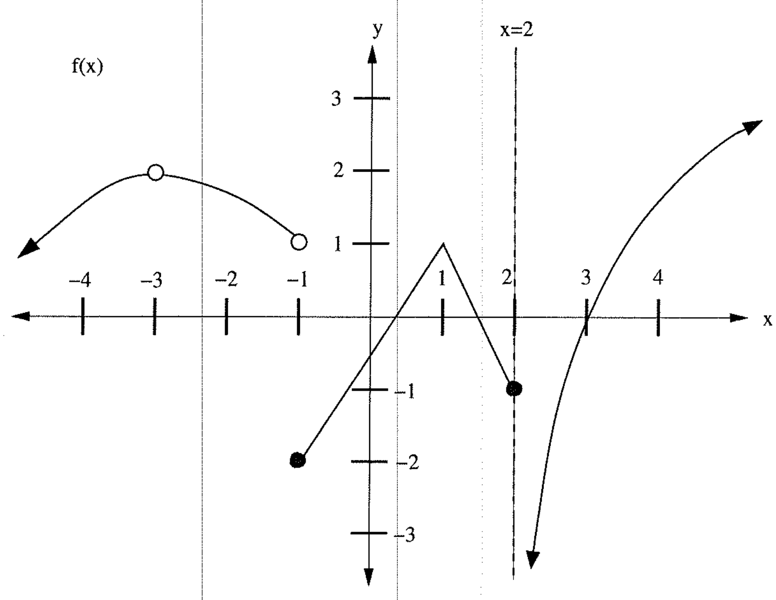


This page has final answers but not full solutions.

7. Determine the intervals of continuity of $g(x) = \frac{\sin x}{x^2 - 5x - 14}$. Use theorems and full sentences to justify your choice.

$(-\infty, -2) \cup (-2, 7) \cup (7, \infty)$

8.



(a) Using the graph above, determine the x values at which f is discontinuous. Then state what type of discontinuity each one is, justifying your answer with limits and/or function values.

$x = -3$, removable ... why?

$x = -1$, jump -- why?

$x = 2$, infinite ... why?

(b) At each point of discontinuity listed in part (a), is f continuous from the right, from the left or neither? Explain.

neither at $x = -3$

from the right at $x = -1$

from the left at $x = 2$

$\Big\}$ Why?

9. WITHOUT actually finding it, show that for all spheres with radii in the interval $[1, 5]$, there is one with a volume of 275 cubic centimeters. (Hint 1: if you forget what the volume of a sphere is, check the front of your textbook. Hint 2: Use something we discussed yesterday!)

Apply the Intermediate Value Theorem !!

10. If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

$f(4) = 3$ and $f'(4) = \frac{1}{4}$