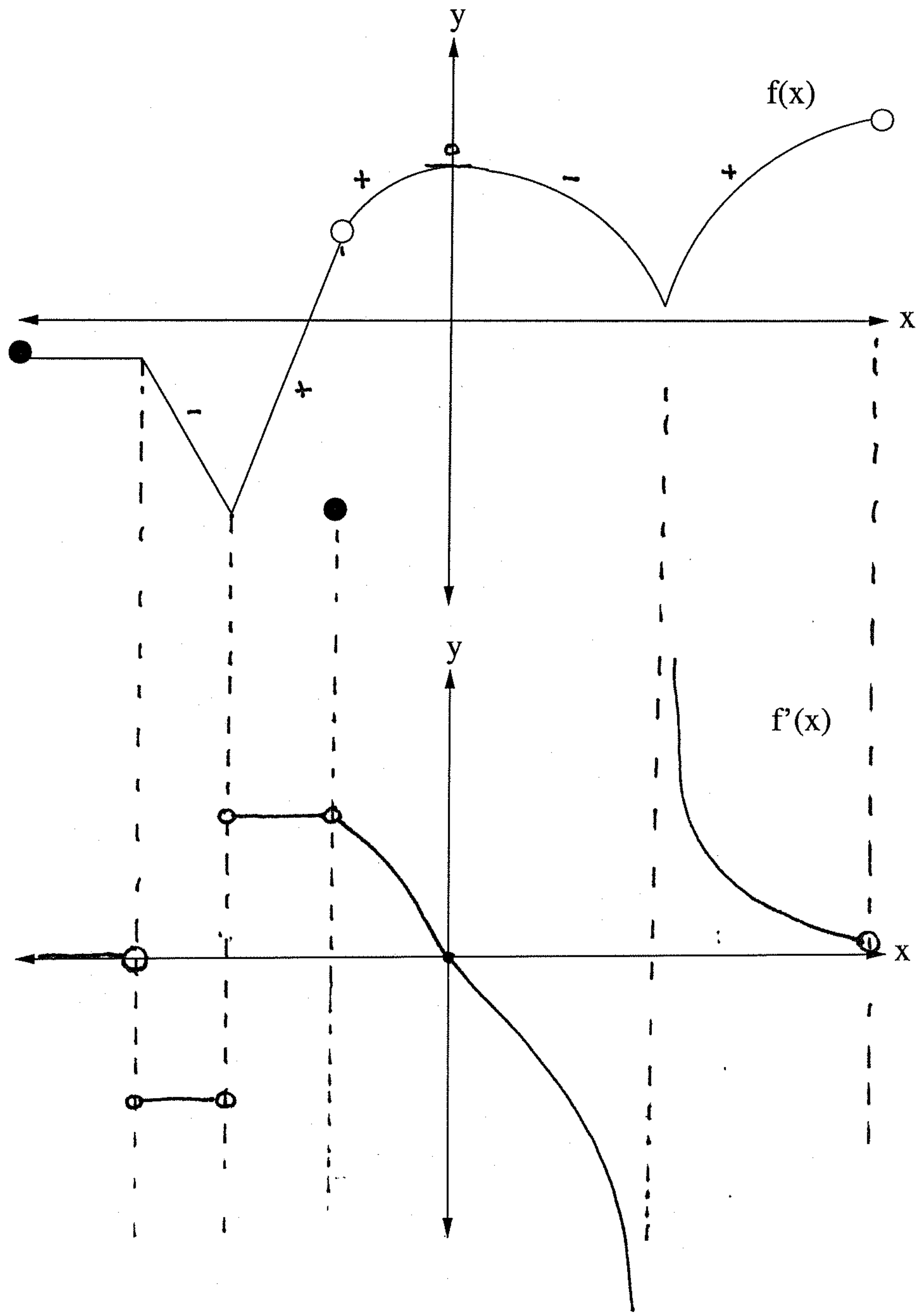
LAB Week 7

MATH 130 Section 2
March 7, 2019
Covering Sections 3.1-3.4

Your Name (Print): ANSWER KEY

1. Given the graph of $y = f(x)$, sketch the graph of $f'(x)$ below.



2. (a) Using the **definition** of the derivative, find a formula for $f'(x)$ if $f(x) = \frac{2x+1}{3-x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(x+h)+1}{3-(x+h)} - \frac{2x+1}{3-x}}{h}$$

$$= \lim_{h\to 0} \frac{[2(x+h)+1][3-x] - [2x+1][3-(x+h)]}{h[3-(x+h)][3-x]}$$

$$= \lim_{h\to 0} \frac{[2x+2h+1][3-x] - [2x+1][3-x-h]}{h[3-x-h][3-x]}$$

$$= \lim_{h\to 0} \frac{(\cancel{6x} - \cancel{2x^2} + 6h - \cancel{2xh} + \cancel{3} - \cancel{x}) - (\cancel{6x} - \cancel{2x^2} - \cancel{2xh} + \cancel{3} - \cancel{x} - h)}{h[3-x-h][3-x]}$$

$$= \lim_{h\to 0} \frac{6h+h}{h[3-x-h][3-x]} = \lim_{h\to 0} \frac{7h}{h[3-x-h][3-x]}$$

$$= \lim_{h\to 0} \frac{7}{[3-x-h][3-x]} = \frac{7}{(3-x)^2}$$

(b) Find the equations of the tangent lines to the curve f that are parallel to the line $y - 7x - 4 = 0$.

parallel to $y = 7x + 4$ means has the same slope, $m = 7$

$$f'(x) = \frac{7}{(3-x)^2} = 7 \Rightarrow 7 = 7(3-x)^2 \Rightarrow (3-x)^2 = 1$$

$$\Rightarrow x^2 - 6x + 9 = 1 \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow (x-4)(x-2) = 0$$

$$\Rightarrow x = 4, \ x = 2$$

$$f(4) = \frac{2(4)+1}{3-4} = \frac{9}{-1} = -9$$

$$f(2) = \frac{2(2)+1}{3-2} = \frac{5}{1} = 5$$

Tangent lines:

$$(y+9) = 7(x-4)$$

$$(y-5) = 7(x-2)$$

3. Evaluate $\lim_{x\to 1} \frac{x^{31}-1}{x-1}$. (Hint: Does this look familiar?) Justify your answer clearly!

This limit is the derivative of $f(x) = x^{31}$ evaluated at $a = 1$.

Thus since $f'(x) = 31x^{30}$, $\lim_{x\to 1} \frac{x^{31}-1}{x-1} = 31(1)^{30} = 31$.

4. Differentiate the following functions and simplify your answers. Explain through work or words.

(a) $f(x) = 7\sqrt[5]{x} - \frac{2}{x^6} + 8x^3 - 9x + 11$ $= 7x^{1/5} - 2x^{-6} + 8x^3 - 9x + 11$

$f'(x) = 7 \cdot \frac{1}{5} x^{-4/5} - 2 \cdot (-6) x^{-7} + 8 \cdot 3x^2 - 9 + 0$

$= \frac{7}{5x^{4/5}} + \frac{12}{x^7} + 24x^2 - 9$

(b) $g(x) = e^3 + \pi^2$

$g'(x) = 0$ since $g(x)$ is a constant and the derivative of a constant is zero

(c) $y = (4x - 3)^2$ $= 16x^2 - 24x + 9$

$\frac{dy}{dx} = 32x - 24$

(d) $h(t) = 7e^t\sqrt{t}$ $= 7e^t(t^{1/2})$

$h'(t) = 7e^t \cdot \frac{1}{2} t^{-1/2} + t^{1/2} \cdot 7e^t$

$= 7e^t \left[\frac{1}{2\sqrt{t}} + \sqrt{t} \right]$

5. Let $f(x)$ be a differentiable function. Consider the function $g(x) = xf(x)$.

(a) Use the **limit definition of the derivative** to find the derivative of the function $g(x) = xf(x)$. Do not use the product rule.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)f(x+h) - xf(x)}{h}$$

$$= \lim_{h\to 0} \frac{xf(x+h) + hf(x+h) - xf(x)}{h}$$

$$= \lim_{h\to 0} \frac{xf(x+h) - xf(x)}{h} + \lim_{h\to 0} \frac{\cancel{h}f(x+h)}{\cancel{h}}$$

$$= x \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} f(x+h)$$

$$= xf'(x) + f(x)$$

(b) **Where did you use the fact that f is continuous? How do you know that f is continuous?**

We used the fact that f is continuous to say that $\lim_{h\to 0} f(x+h) = f(x)$,

i.e. so we could just plug in 0 for h.

(c) Check your answer using the product rule for derivatives.

$$g'(x) = x \cdot f'(x) + f(x) \cdot 1$$

$$= xf'(x) + f(x) \quad \checkmark$$

6. A particle moves along a straight line so that its position is $s(t) = t^3 - 9t^2 + 15t$, where t is measured in seconds and s in meters.

(a) Determine the velocity and acceleration (the rate of change in velocity).

$$v(t) = 3t^2 - 18t + 15$$

$$a(t) = 6t - 18$$

(b) When is the particle at rest?

The particle is at rest when $v(t) = 0$.

$3t^2 - 18t + 15 = 0 \Rightarrow 3(t^2 - 6t + 5) = 0 \Rightarrow 3(t-5)(t-1) = 0$

Thus the particle is at rest when $t = 5$ and $t = 1$.

(c) When is there no acceleration?

There is no acceleration when $a(t) = 0$.

$6t - 18 = 0 \Rightarrow 6(t-3) = 0$

Thus there is no acceleration when $t = 3$.

ANSWERS NOT SOLUTIONS

7. (a) Turn to page 164 in your textbook. Check out how we can use the Product Rule to prove the Quotient Rule! Isn't that neat! Write down the proof here and ask if you have any questions for any of the steps. Fill in details to explain anywhere the book leaves out explanations.

* see page 164 * :)

(b) Find the derivative of $h(t) = \dfrac{t^3 - 6t^2 + 8t}{t-2}$ using the Quotient Rule.

$h'(t) = 2t - 4, \quad t \neq 2$

(c) Find the derivative of $h(t) = \dfrac{t^3 - 6t^2 + 8t}{t-2}$ withOUT using the Quotient Rule. Which way do you prefer?

same as (b) :)

8. Find two functions $f(x)$ and $g(x)$ for each function $h(x)$ below, such that $h(x) = (f \circ g)(x)$ and neither f nor g is trivial (i.e. neither $f(x)$ nor $g(x)$ is just x).

(a) $h(x) = \sin 3x$

$f(x) = \sin x$

$g(x) = 3x$

(b) $h(x) = \sec^2 x$

$f(x) = x^2$

$g(x) = \sec x$

9. The limit $\lim_{h \to 0} \dfrac{3(2+h)^2 - 5 - 7}{h}$ represents the derivative of a function f at a number a. Determine $f(x)$ and the value of a.

$f(x) = 3x^2 - 5$

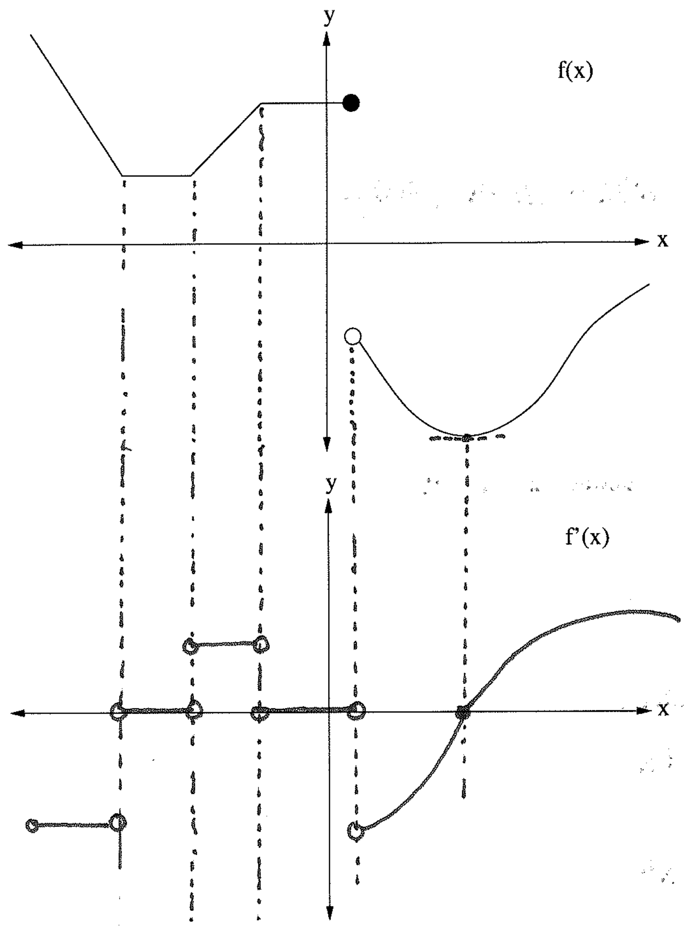
$a = 2$

10. There are two tangent lines to the curve $y = 4x - x^2$ that pass through (2,5). Find an equation for each of the lines.

$y = -2x + 9$

$y = 2x + 1$

11. Given the graph of $y = f(x)$, sketch the graph of $f'(x)$ below.



12. Let $f(x) = 40\sqrt{x} - x$. Find the all points on the graph of f at which the tangent line is parallel to the line $3x - 2y = -10$.

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