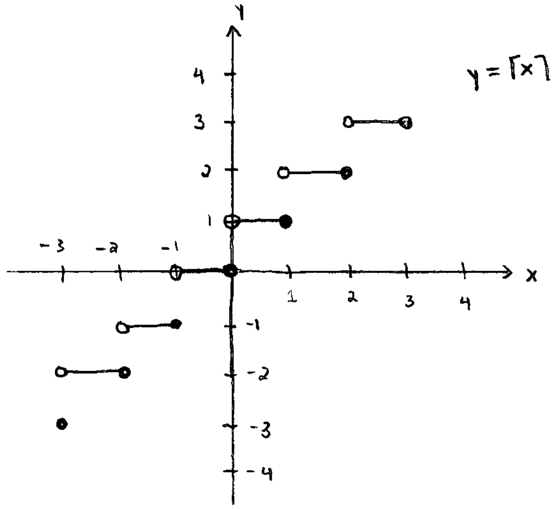
MATH 130 : Main Exercises Week 4

① (a)



(b) (i) $\lim_{x \to -2^-} \lceil x \rceil = -2$ (ii) $\lim_{x \to -1.5} \lceil x \rceil = -1$ (iii) $\lim_{x \to 2^+} \lceil x \rceil = 3$

(c) (i) $\lim_{x \to a^-} \lceil x \rceil = a$ since any $x > a-1$ and $x < a$ will have $\lceil x \rceil = a$.

(ii) $\lim_{x \to a^+} \lceil x \rceil = a+1$ since any $x > a$ and $x < a+1$ will have $\lceil x \rceil = a+1$.

(d) If a is an integer, then $\lim_{x \to a} \lceil x \rceil$ does not exist since the right hand and left hand limits will be different. For all other values of a, $\lim_{x \to a} \lceil x \rceil$ exist since the function close to a will look like a line.

② $\lim_{x \to -4} \left(\frac{x}{4} + 7 \right) = -\frac{4}{4} + 7 = 6$

Proof: Given $\epsilon > 0$, let $\delta = 4\epsilon$.

Then if $0 < |x+4| < \delta$, then

$$\begin{aligned} \left| \left(\tfrac{x}{4} + 7 \right) - 6 \right| &= \left| \tfrac{x}{4} + 1 \right| \\ &= \left| \tfrac{1}{4}(x+4) \right| \\ &= \left| \tfrac{1}{4} \right| |x+4| \\ &= \tfrac{1}{4} |x+4| \\ &< \tfrac{1}{4} \delta = \tfrac{1}{4}(4\epsilon) = \epsilon. \end{aligned}$$

Thus $\lim_{x \to -4} \left(\frac{x}{4} + 7 \right) = 6$.

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