WEEK 15 LAB KEY

MATH 131 Sections 2 and 3
December 10, 2015 Your Name (Print): ____________________

This lab will not be collected!

Covering Sections 8.3-9.2 and a review of some topics from the first half of the semester

This is a list of problems and it is not expected that enough room is provided for you to do all of them here. It is not necessary to do them in order, choose on which section you would like to focus. **Please bring this to class tomorrow as well!**

Part 1: Power Series

1. Consider the function $f(x) = \sqrt{x}$.

(a) Determine the degree three Taylor polynomial, p_3, for f centered at $a = 4$.

$p_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

(b) Use your work in (a) to determine a general order n Taylor polynomial, p_n for f.

This is more complicated than I intended! $p_n(x) = 2 + \frac{1}{4}(x-4)\sum_{k=2}^{\infty} \frac{(-1)^{k+1}(1*3*5*\cdots*(2k-3))}{2^{3k-1}k!}(x-4)^k$

(c) Use p_3 to approximate a value for $\sqrt{3.9}$.

$p_3(3.9) = 1.974841797$

2. (We will talk about these tomorrow, so do NOT do them now!) Determine the radius and interval of convergence for the following power series:

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$R = 1$; $I = (-1, 1]$
Notes: use the Ratio Test to find the open interval and then plug in the endpoints to the series to determine if the series converges for those values.

(b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$R = \infty$; $I = (-\infty, \infty)$

(c) $\sum_{n=0}^{\infty} \frac{x^n}{3n^2+1}$

$R = 1$; $I = [-1, 1]$

(d) $\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n+1}$

$R = \frac{1}{2}$; $I = [-\frac{1}{2}, \frac{1}{2})$

Part 2: Series

3. Determine whether the following series are convergent or divergent. If they are convergent, find the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left[2\left(\frac{3}{5}\right)^n+3\left(\frac{4}{9}\right)^n\right]$

   This series converges to $\dfrac{27}{5}$, using geometric series and the theorem about sums of series.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^2+4}}$

   This series is divergent by the Test for Divergence.

4. Determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=3}^{\infty}\frac{n^2}{e^n}$

   This series is convergent.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n(n+1)(n+2)}}$

   This series is divergent.

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n\ln n}{(n+1)^3}$

   This series is convergent.

5. Determine whether the following series converge absolutely, conditionally, or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n2^{4n}}{(2n+1)!}$

   This series is absolutely convergent.

   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{\frac{1}{n}}}{n}$

   This series is conditionally convergent.

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)4n^3+1}{n^4}$

   This series is conditionally convergent.

Part 3: True/False and Multiple Choice Questions

The final exam will have both True/False and Multiple Choice questions on it. When answering the True/False questions below, be sure to understand why your answer is correct and think about how you might reword the statement to have a different answer. Most of the Multiple Choice questions are regular problems. Work out your solution and find it among the choices given.

6. **True/False**

Circle T if the statement is **always** true or F if the statement is not always true.

T **F** $\int_a^b f(x)dx$ represents the area under the graph of f from $x = a$ to $x = b$.

T **F** If the sequence $\{a_n\}$ converges, then the sequence $\{(-1)^n a_n\}$ converges.

T F $\int_{-3}^{3} \frac{x^5}{\sqrt{12 - x^2}} dx = 0$

T **F** If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

7. **Multiple Choice**

Circle the number which best answers the question or completes the statement.

(A) If $g(x) = \int_{\sqrt{x}}^{2} \frac{t}{\sqrt{t^4 + 7}} dt$, then $g'(x) =$

(i) $\frac{\sqrt{x}}{\sqrt{x^2 + 7}}$

(ii) $-\frac{\sqrt{x}}{\sqrt{x^2 + 7}}$

(iii) $\frac{1}{2\sqrt{x^2 + 7}}$

(iv) $-\frac{1}{2\sqrt{x^2 + 7}}$

(v) none of the above

(B) For what values of c does the series $\sum_{n=1}^{\infty} (c - 1)^n$ converge?

(i) $-2 < c < 0$

(ii) $-2 < c \leq 0$

(iii) $-1 < c < 1$

(iv) $-1 < c \leq 1$

(v) $0 < c \leq 2$

(vi) $0 < c < 2$

Part 4: Revisiting some old friends

8. Solve the differential equation: $f''(x) = x + \sqrt{x}$, $f(1) = 1$, $f'(1) = 2$.

$$f(x) = \frac{x^3}{6} + \frac{4}{15}x^{\frac{5}{2}} + \frac{5}{6}x - \frac{4}{15}$$

9. Let $f(x) = 1 + 4x^2$ on the interval $[0, 3]$. Use six rectangles to estimate the area under the curve using right endpoints. Show a diagram with the rectangles. How accurate is your estimation?

I will let you draw the pretty picture! Then find $R_6 = \frac{1}{2}[f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)] = \cdots$. Finally, check your accuracy with an integral!

10. Use the **definition** of area to find the area under the graph $g(x) = 3x^2 + 7$ on the interval $[-1, 7]$. How can you check your answer?

$A = 400$; Remember after you use the definition, you can check your work using short-cuts!

11. Evaluate $\displaystyle\int_3^5 \frac{4}{x^2 - 4x + 3}dx$.

This is a divergent improper integral!

12. Sketch the region enclosed by $x + y^2 = 2$ and $x + y = 0$. Find the area of this region.

$A = \frac{9}{2}$

13. Sketch the region enclosed by $y = \cos x$ and $y = \cos^2 x$ between $x = 0$ and $x = \pi$. Find the area of this region.

$A = 2$

14. Sketch the region enclosed by $y = x^3$, $y = 2x + 4$ and $x = 0$.

(a) Find the volume of the solid generated by rotating the region about the x-axis.

$V = \frac{1184\pi}{21}$

(b) find the volume of the solid generated by rotating the region about the line $x = 3$.

$V = \frac{512\pi}{15}$

15. Find the arc length of $y = \frac{1}{2}x^2$ over the interval $[0, 4]$.

$L = \frac{1}{2}[4\sqrt{17} + \ln(4 + \sqrt{17})]$

16. Evaluate $\displaystyle\int x \arctan x \, dx$.

$\frac{x^2}{2} \arctan x - \frac{1}{2}x + \frac{1}{2} \arctan x + C$ Don't forget your family!

17. Suppose that 10 in-lb are needed to stretch a spring from its natural length of 20 inches to a length of 25 inches. How much work is needed to stretch it from 25 to 27 inches?

$W = \frac{48}{5}$ in-lbs