WEEK 3 LAB

MATH 131 Section 2
September 13, 2018
Covering Sections 5.2-5.4

Your Name (Print): ANSWER KEY

1. Suppose $\int_1^7 f(x)dx = 28$ and $\int_4^1 f(x)dx = 16$. Evaluate the following integrals being careful to **show each step**. Each step should be the result of **ONE** of the properties of definite integrals as stated in Table 5.4 in Section 5.2.

(a) $\int_4^7 f(x)dx$

$$\int_1^4 f(x)dx + \int_4^7 f(x)dx = \int_1^7 f(x)dx, \quad \text{so} \quad \int_4^7 f(x)dx = \int_1^7 f(x)\,dx - \int_1^4 f(x)dx$$

$$\text{and thus} \quad \int_4^7 f(x)dx = \int_1^7 f(x)dx + \int_4^1 f(x)dx = 28 + 16 = 44$$

(b) $\int_7^4 f(x)dx = -\int_4^7 f(x)dx = -44$

(c) $\int_4^7 [5 - 3f(x)]dx = \int_4^7 5dx - \int_4^7 3f(x)dx$

$$= \int_4^7 5dx - 3\int_4^7 f(x)dx$$

$$= 5(7-4) - 3(44)$$

$$= 15 - 132$$

$$= -117$$

2. Evaluate the following. **Explain your work.**

(a) $\frac{d}{dx}\int_0^{\frac{\pi}{2}} \sin^3 x\,dx$ $= 0$

since the definite integral is a number when evaluated and the derivative of a number is zero

(b) $\int_0^{\frac{\pi}{4}} \left(\frac{d}{dx}\sin^3 x\right) dx$ $= \sin^3 x \Big]_0^{\pi/4} = \left[\sin\frac{\pi}{4}\right]^3 - \left[\sin 0\right]^3$

$= \left(\frac{\sqrt{2}}{2}\right)^3 - 0$

$= \frac{2\sqrt{2}}{8}$

$= \frac{\sqrt{2}}{4}$

(c) $\frac{d}{dx}\int_x^1 \sin^3 t\,dt$ $= \frac{d}{dx}\left[-\int_1^x \sin^3 t\,dt\right]$

$= -\sin^3 x$

(d) $\int \left(\frac{d}{dx}\sin^3 x\right) dx$ $= \sin^3 x + C$ by the Fundamental Theorem of Calculus

3. If $g(x) = \int_{x^3}^{e^x} t^2 \cos t \, dt$, find $g'(x)$. Show each step.

$$g(x) = \int_{x^3}^{0} t^2 \cos t \, dt + \int_{0}^{e^x} t^2 \cos t \, dt$$

$$= -\int_{0}^{x^3} t^2 \cos t \, dt + \int_{0}^{e^x} t^2 \cos t \, dt$$

Thus.

$$g'(x) = (e^x)^2 \cos(e^x) \cdot e^x - (x^3)^2 \cos(x^3) \cdot 3x^2$$

$$= e^{3x} \cos(e^x) - 3x^8 \cos(x^3)$$

4. Find the average value of $f(x) = |4 - x|$ on $[0, 6]$ **and** determine the points c where the average occurs. How do we know that such c must exist? Note that you can solve your definite integral in two ways. Be sure to show your work whichever way you choose.

$$f(x) = |4-x| = \begin{cases} 4-x & \text{if } x \leq 4 \\ x-4 & \text{if } x > 4 \end{cases}$$

$$\bar{f} = \frac{1}{6-0} \int_0^6 |4-x| \, dx = \frac{1}{6}\left[\int_0^4 (4-x)\,dx + \int_4^6 (x-4)\,dx \right]$$

$$= \frac{1}{6}\left[\left[4x - \frac{x^2}{2}\right]_0^4 + \left[\frac{x^2}{2} - 4x\right]_4^6 \right]$$

$$= \frac{1}{6}\left[\left(16 - \frac{16}{2}\right) - 0 + \left[\left(\frac{36}{2} - 24\right) - \left(\frac{16}{2} - 16\right)\right] \right]$$

$$= \frac{1}{6}\left[8 - 6 + 8\right] = \frac{1}{6}[10] = \frac{5}{3}$$

$|4-x| = \frac{5}{3}$ if $4 - x = \frac{5}{3}$ or $-(4-x) = \frac{5}{3}$

$\frac{4(3)-5}{3} = x$ or $x = \frac{5+4(3)}{3}$

$x = \frac{7}{3}$ or $x = \frac{17}{3}$ (both in interval !)

So the average occurs at $c = \frac{7}{3}$ and $c = \frac{17}{3}$.

We know that such a c exists by the Mean Value Theorem for Integrals.

5. Evaluate the following integrals. Be sure to show each step or explain your process. BE CAREFUL!

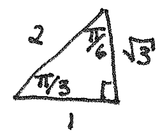
(a) $\displaystyle\int_1^4 \frac{(x-2)^2}{\sqrt{x}}\,dx = \int_1^4 \frac{x^2-4x+4}{\sqrt{x}}\,dx = \int_1^4 (x^{3/2}-4x^{1/2}+4x^{-1/2})\,dx$

$$= \frac{2}{5}x^{5/2} - 4\cdot\frac{2}{3}x^{3/2} + 4\cdot 2x^{1/2}\Big]_1^4$$

$$= \left[\left(\frac{2}{5}(4)^{5/2} - \frac{8}{3}(4)^{3/2} + 8\sqrt{4}\right) - \left(\frac{2}{5} - \frac{8}{3} + 8\right)\right]$$

$$= \frac{64}{5} - \frac{64}{3} + 16 - \frac{2}{5} + \frac{8}{3} - 8 = \frac{62}{5} - \frac{56}{3} + 8 = \frac{186-280+120}{15} = \frac{26}{15}$$

(b) $\displaystyle\int_{\sqrt{2}}^2 \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x\Big]_{\sqrt{2}}^{2}$



$\sec\theta = 2$
$\cos\theta = \frac{1}{2}$

$$= \operatorname{arcsec} 2 - \operatorname{arcsec}\sqrt{2}$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$$

(c) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = \tan x\Big]_{-\frac{\pi}{4}}^{\pi/4}$

$$= \tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)$$

$$= 1 - (-1)$$

$$= 2$$

(d) $\displaystyle\int_{-1}^1 \frac{3}{x^4}\,dx$ DNE since there is an infinite discontinuity at $x=0$

ANSWERS NOT SOLUTIONS FOR 6 & 7

6. The phases of the moon occur in a regular, predictable 28 day lunar cycle. Data (for Boston) from The Old Farmers Almanac indicate that the hours, $v(x)$, the moon is visible per day over the course of the cycle (assuming a cloudless sky) is $v(x) = 2.615\cos\left(\frac{\pi}{14}x - \pi\right) + 12.435$, where x is the day of the cycle. Find the average number of hours the moon is visible per day in Boston using calculus. Why does your answer make sense?

12.435 hours per day on average

7. Evaluate the following integrals. Be sure to show each step or explain your process. BE CAREFUL (that is, think carefully about your intervals)!

(a) $\int_0^{\pi} \sec^2 x \, dx$ DNE ... why?

(b) $\int_{-1}^{3} |2x - x^2| dx = 4$