WEEK 5 LAB

MATH 131 Section 2
September 27, 2018
Covering Sections 6.1-6.3

Your Name (Print): KEY

1. Suppose that the velocity of an object measured in meters/sec is $v(t) = t^2 - 6t + 8$ for $0 \le t \le 5$.

(a) Find the displacement (net change in position) over the given interval.

$$\text{Displacement} = \int_0^5 (t^2 - 6t + 8)\,dt = \frac{t^3}{3} - 3t^2 + 8t\Big]_0^5 = \left(\frac{125}{3} - 75 + 40\right) - 0 = \frac{20}{3} \text{ meters}$$

(b) Find the TOTAL distance traveled over the same interval.

$$\text{Distance} = \int_0^5 |v(t)|\,dt = \int_0^2 (t^2 - 6t + 8)\,dt + \int_2^4 -(t^2 - 6t + 8)\,dt$$

$$+ \int_4^5 (t^2 - 6t + 8)\,dt = \cdots = \frac{28}{3} \text{ m}$$

$t^2 - 6t + 8 = (t-4)(t-2)$

		2		4	
$v(t)$	+ + +	0	− − −	0	+ + +
$t-4$	− − −		− − −	0	+ +
$t-2$	− −	0	+ + +		+ +

Thus $|v(t)| = \begin{cases} t^2 - 6t + 8 & \text{if } t \le 2 \\ -(t^2 - 6t + 8) & \text{if } 2 < t \le 4 \\ t^2 - 6t + 8 & \text{if } t > 4 \end{cases}$

2. When records were first kept, the population of a rural town was 250 people ($t = 0$). Since then the population has grown at a rate $P'(t) = 30(1 + \sqrt{t})$, where t is measured in years.

(a) What was the population at time $t = 20$ years?

$$P(20) = P(0) + \int_0^{20} 30(1 + \sqrt{t})\,dt = 250 + \left(30\left(t + \frac{2}{3}t^{3/2}\right)\Big]_0^{20}\right)$$

$$= 250 + 30\left(\left(20 + \frac{2}{3}(20)^{3/2}\right) - 0\right) = 250 + 30\left(20 + \frac{2}{3}(2\sqrt{5})^3\right)$$

$$= 250 + 600 + 20 \cdot 8 \cdot 5\sqrt{5} = 850 + 800\sqrt{5} \approx 850 + 1788.9 \text{ so } \approx 2639 \text{ people}$$

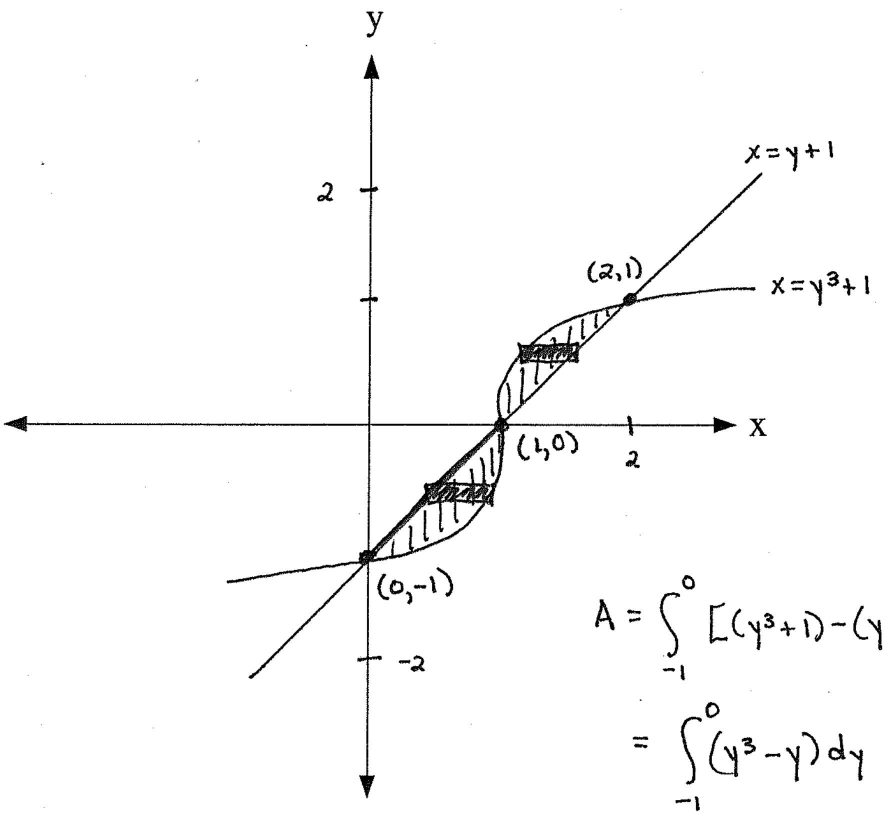
(we must have whole people ☺)

(b) Find the population $P(t)$ for an arbitrary time $t \ge 0$.

$$P(t) = P(0) + \int_0^t 30(1 + \sqrt{x})\,dx = 250 + \left[30\left(x + \frac{2x^{3/2}}{3}\right)\right]_0^t$$

$$= 250 + 30\left(t + \frac{2}{3}t^{3/2}\right) \text{ people}$$

3. Find the area of the region bounded by $x = y^3 + 1$ and $x = y + 1$.



$$y^3+1 = y+1$$
$$y^3 - y = 0$$
$$y(y^2-1) = 0$$
$$y(y+1)(y-1) = 0$$
$$y=0,\ y=-1,\ y=1$$
$$(1,0)\,;\,(0,-1)\,;\,(2,1)$$

$$A = \int_{-1}^{0} [(y^3+1)-(y+1)]\,dy + \int_{0}^{1} [(y+1)-(y^3+1)]\,dy$$

$$= \int_{-1}^{0} (y^3-y)\,dy + \int_{0}^{1} (y-y^3)\,dy$$

$$= \frac{y^4}{4} - \frac{y^2}{2}\Big]_{-1}^{0} + \frac{y^2}{2} - \frac{y^4}{4}\Big]_{0}^{1}$$

$$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right]$$

$$= 1 - \frac{1}{2}$$

$$= \boxed{\frac{1}{2}}$$

4. Find the area of the region bounded by $y = e^x$, $x + 3y = 3$, and $x = 3$. (Leave e's in your answer.)

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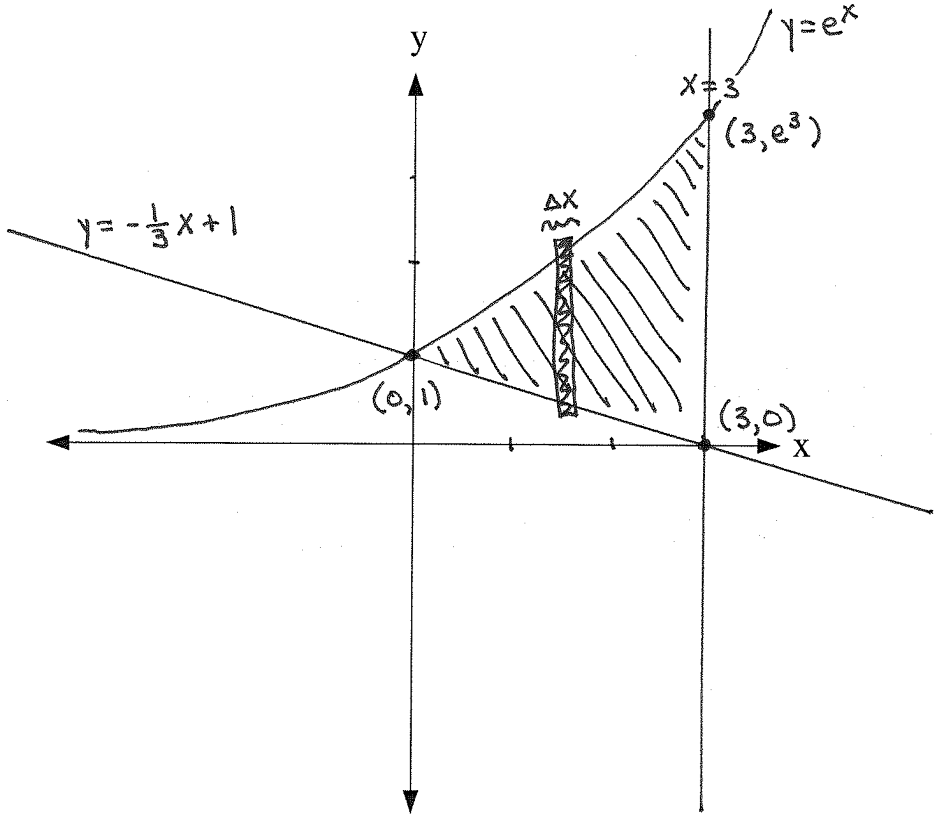


$$A = \int_0^3 \left(e^x - \left(-\tfrac{1}{3}x + 1\right)\right) dx$$

$$= \int_0^3 \left(e^x + \tfrac{1}{3}x - 1\right) dx$$

$$= e^x + \tfrac{1}{3} \cdot \tfrac{1}{2}x^2 - x \Big]_0^3 \quad = e^x + \tfrac{1}{6}x^2 - x \Big]_0^3$$

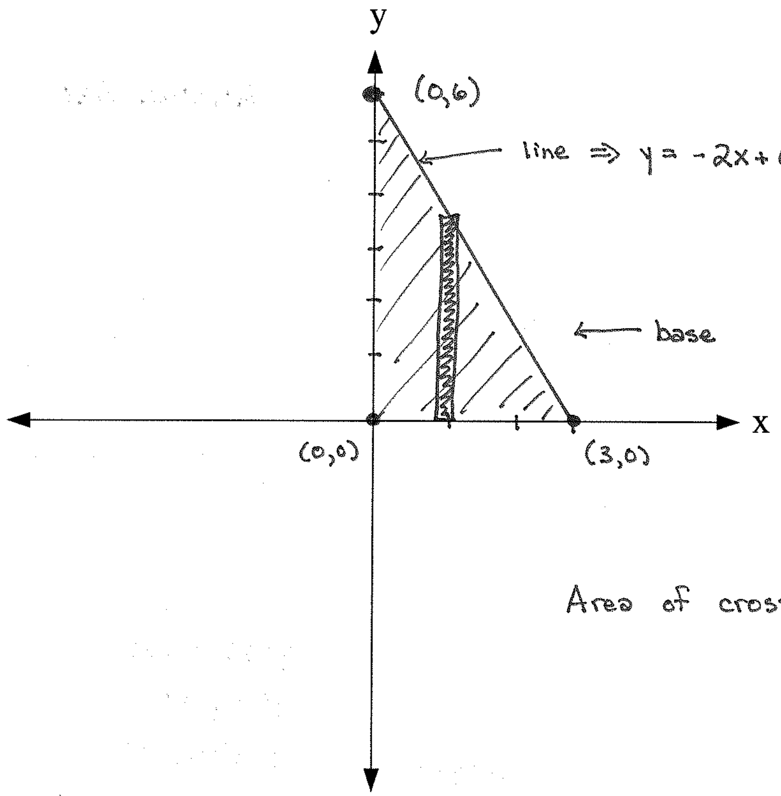
$$= \left(e^3 + \tfrac{9}{6} - 3\right) - \left(e^0 + \tfrac{0}{6} - 0\right)$$

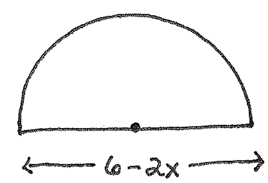
$$= e^3 + \tfrac{3}{2} - 3 - 1$$

$$= e^3 + \tfrac{3}{2} - 4 \quad = e^3 + \tfrac{3}{2} - \tfrac{8}{2}$$

$$= \boxed{e^3 - \tfrac{5}{2}} \qquad \approx 17.5855$$

5. Find the volume of the solid whose base is the triangle with vertices $(0,0)$, $(3,0)$ and $(0,6)$, and whose cross sections perpendicular to the base and parallel to the y-axis are semicircles.

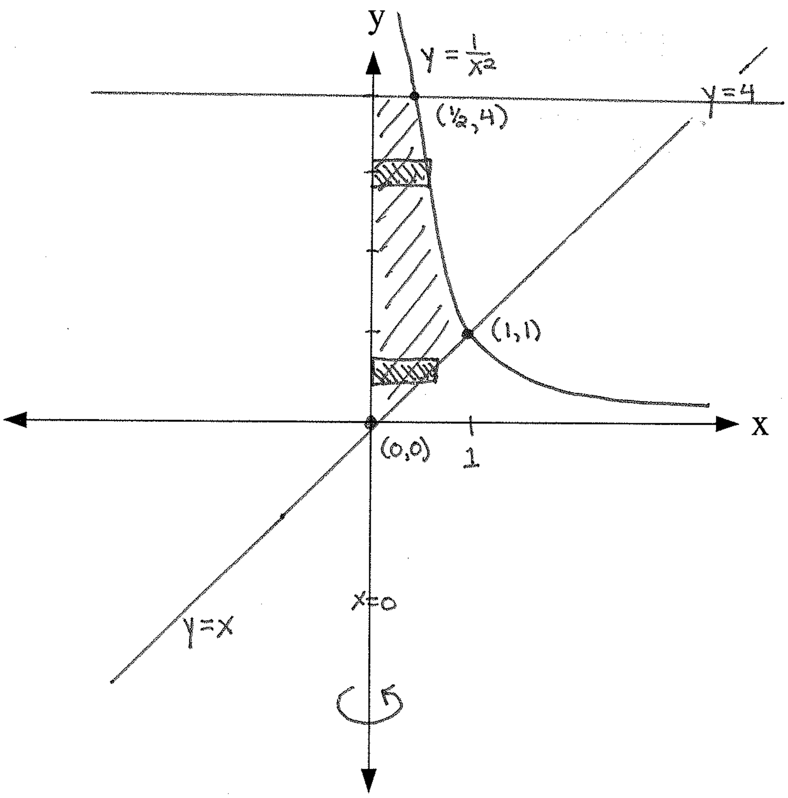




$$\text{Area of cross section} = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi\left[\frac{6-2x}{2}\right]^2$$
$$= \frac{\pi}{2}\left[9 - 6x + x^2\right]$$

$$V = \int_0^3 A(x)\,dx$$
$$= \int_0^3 \frac{\pi}{2}\left[9 - 6x + x^2\right]dx$$
$$= \frac{\pi}{2}\left[9x - 3x^2 + \frac{x^3}{3}\right]_0^3$$
$$= \frac{\pi}{2}\left[\left(\cancel{27} - \cancel{27} + \frac{27}{3}\right) - 0\right]$$
$$= \boxed{\frac{9\pi}{2}}$$

6. Find the volume of the solid obtained by revolving the region bounded by the curves $y = \dfrac{1}{x^2}$, $y = x$, $x = 0$ and $y = 4$ about the y-axis.



$$y = \frac{1}{x^2} \Rightarrow x = \pm \frac{1}{\sqrt{y}}$$

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$$V = \int_0^1 \pi y^2\,dy + \int_1^4 \pi \left(\frac{1}{\sqrt{y}}\right)^2 dy$$

$$= \pi \int_0^1 y^2\,dy + \pi \int_1^4 \frac{1}{y}\,dy$$

$$= \pi \left[\frac{y^3}{3}\right]_0^1 + \pi \ln|y| \Big]_1^4$$

$$= \pi \left[\frac{1}{3} - 0\right] + \pi \left[\ln 4 - \overset{0}{\cancel{\ln 1}}\right]$$

$$= \pi \left[\frac{1}{3} + \ln 4\right]$$

Shell :

$$V = \int_0^{1/2} 2\pi x (4 - x)\,dx + \int_{1/2}^1 2\pi x \left(\frac{1}{x^2} - x\right) dx$$

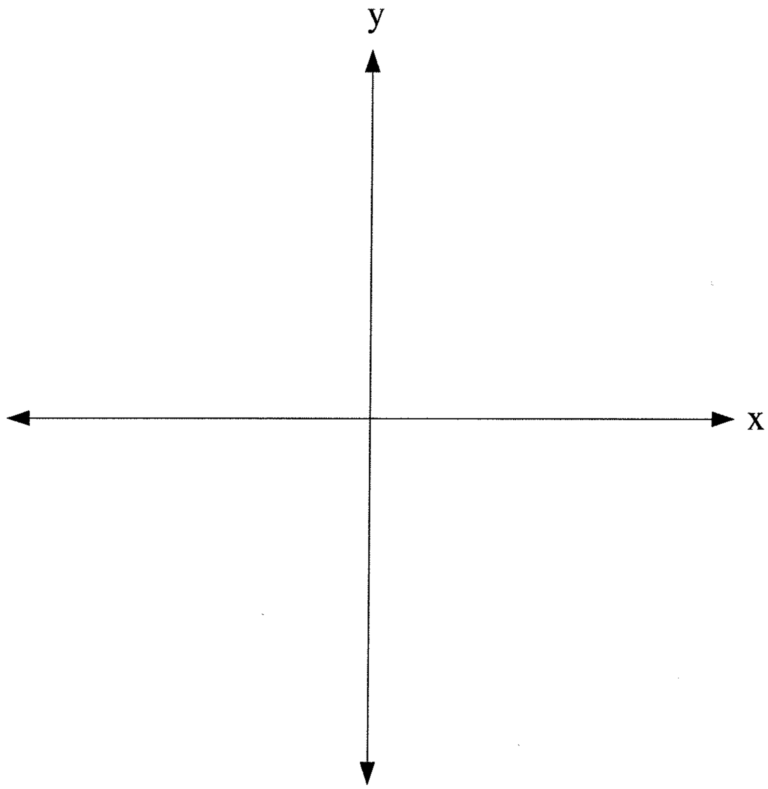
$$= 2\pi \int_0^{1/2} (4x - x^2)\,dx + 2\pi \int_{1/2}^1 \left(\frac{1}{x} - x^2\right) dx$$

$$= 2\pi \left[2x^2 - \frac{x^3}{3}\right]_0^{1/2} + 2\pi \left[\ln|x| - \frac{x^3}{3}\right]_{1/2}^1$$

$$= 2\pi \left[2\left(\frac{1}{4}\right) - \cancel{\frac{1}{3}\left(\frac{1}{8}\right)} - 0\right] + 2\pi \left[\ln 1 - \frac{1}{3} - \ln\left|\frac{1}{2}\right| + \cancel{\frac{1}{3}\left(\frac{1}{8}\right)}\right]$$

$$= 2\pi \left[\frac{1}{2} - \frac{1}{3}\right]$$

7. Find the volume of the solid obtained by revolving the region bounded by the curves $y = \frac{1}{x^2}$, $y = x$, $y = 0$ and $x = 3$ about the x-axis.



ANS: $\frac{53}{81}\pi$

8. Evaluate the following integrals. (Hint: Complete the square!)

(a) $\displaystyle\int \frac{1}{(x+7)\sqrt{4x^2+56x+180}}dx$

ANS: $\frac{1}{4}\operatorname{arcsec}\left(\frac{x+7}{2}\right) + C$

(b) $\displaystyle\int \frac{1}{x^2+6x+16}dx$

ANS: $\frac{1}{\sqrt{7}}\arctan\left(\frac{x+3}{\sqrt{7}}\right) + C$