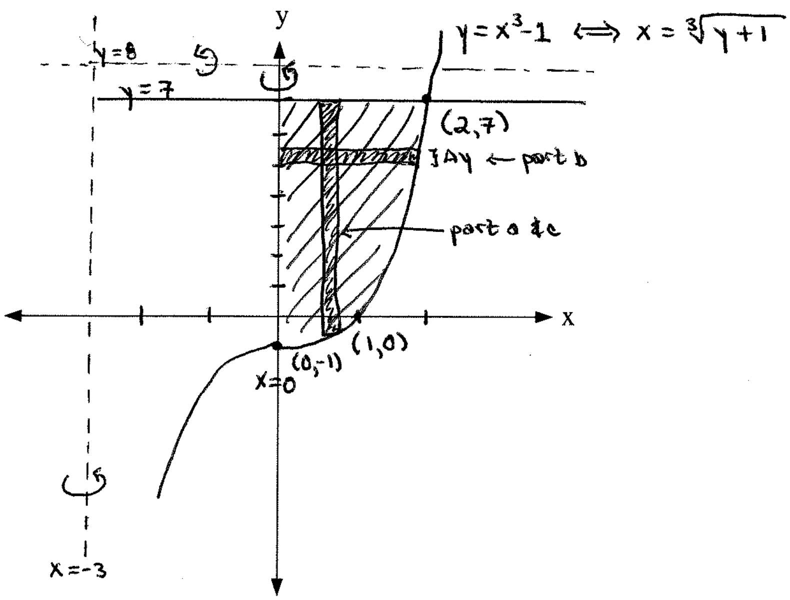
WEEK 6 LAB

MATH 131 Section 2
October 4, 2018
Covering Sections 6.3-6.5

Your Name (Print): KEY

1. Sketch the region bounded by the curves $y = x^3 - 1$, $y = 7$ and $x = 0$.



(a) Set up an integral for finding the volume of the solid obtained by revolving the region about the $y-axis$, but do not evaluate it. Include an estimating rectangle in your diagram above, and state which method you chose to use.

$$V = \int_0^2 2\pi x\,(7 - (x^3 - 1))\,dx \quad ; \text{ shell method}$$

(b) Set up an integral for finding the volume of the solid obtained by revolving the region about $x = 8$, but do not evaluate it. Include an estimating rectangle in your diagram above, and state which method you chose to use.

$$V = \int_{-1}^{7} 2\pi (8-y)(\sqrt[3]{y+1})\,dy \quad ; \text{ shell method}$$

(c) Find the volume of the solid obtained by revolving the region about the line $x = -3$. That is, set up AND evaluate it! Include an estimating rectangle in your diagram above, and state which method you chose to use.

$$V = \int_0^2 2\pi (3+x)(7 - (x^3-1))\,dx = \int_0^2 2\pi (3+x)(8 - x^3)\,dx$$

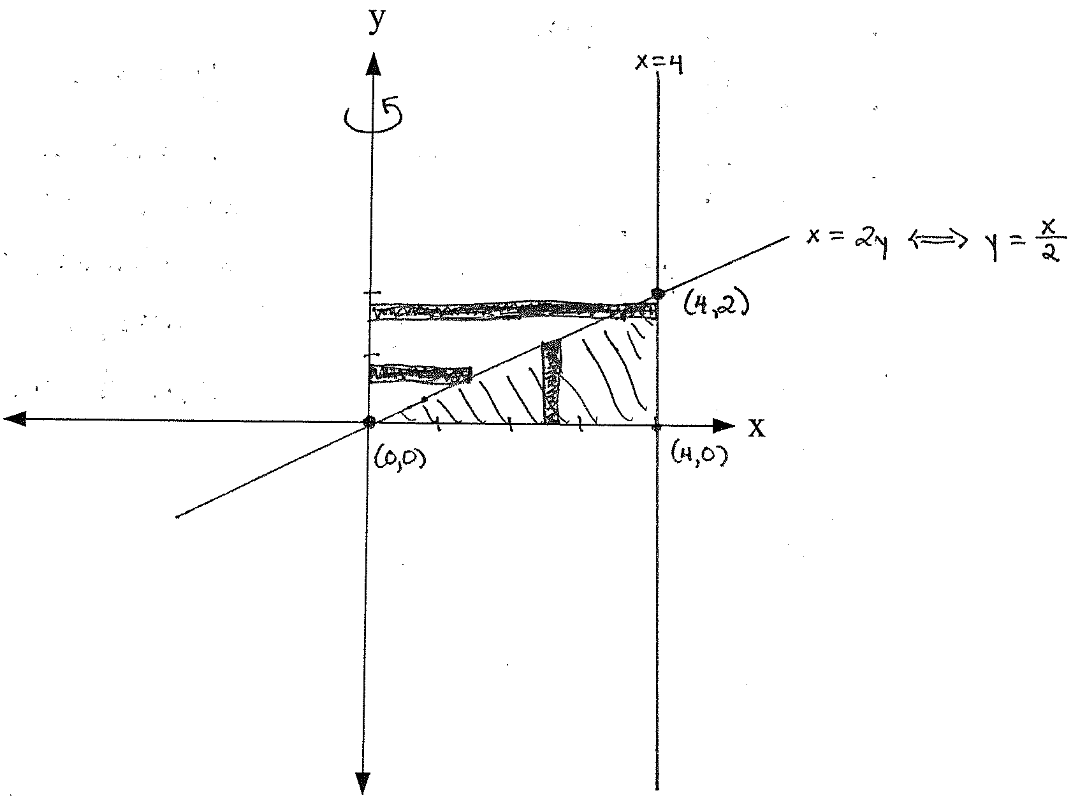
$$= 2\pi \int_0^2 (24 - 3x^3 + 8x - x^4)\,dx = 2\pi \left[24x - \tfrac{3}{4}x^4 + 4x^2 - \tfrac{1}{5}x^5\right]_0^2$$

$$= 2\pi \left[\left(48 - 12 + 16 - \tfrac{32}{5}\right) - 0\right] = 2\pi \left[52 - \tfrac{32}{5}\right]$$

$$= 2\pi \left[\frac{260 - 32}{5}\right] = \frac{456\pi}{5}$$

2. Give a geometric argument that explains why the following integrals have equal values. Use both a sketch and **full sentences** for a complete argument. Think about what question might have been asked for which these integrals could be part of the solution.

$$\pi \int_0^2 [16 - (2y)^2]dy = 2\pi \int_0^4 x\left(\frac{x}{2}\right)dx$$



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Consider the region bounded by $x = 2y$, $x = 4$ and $y = 0$. Rotate this region about the y-axis. To find the volume of the resulting solid, we would consider a disk of radius four minus a disk of radius $2y$ as y varies from zero to two, giving us:

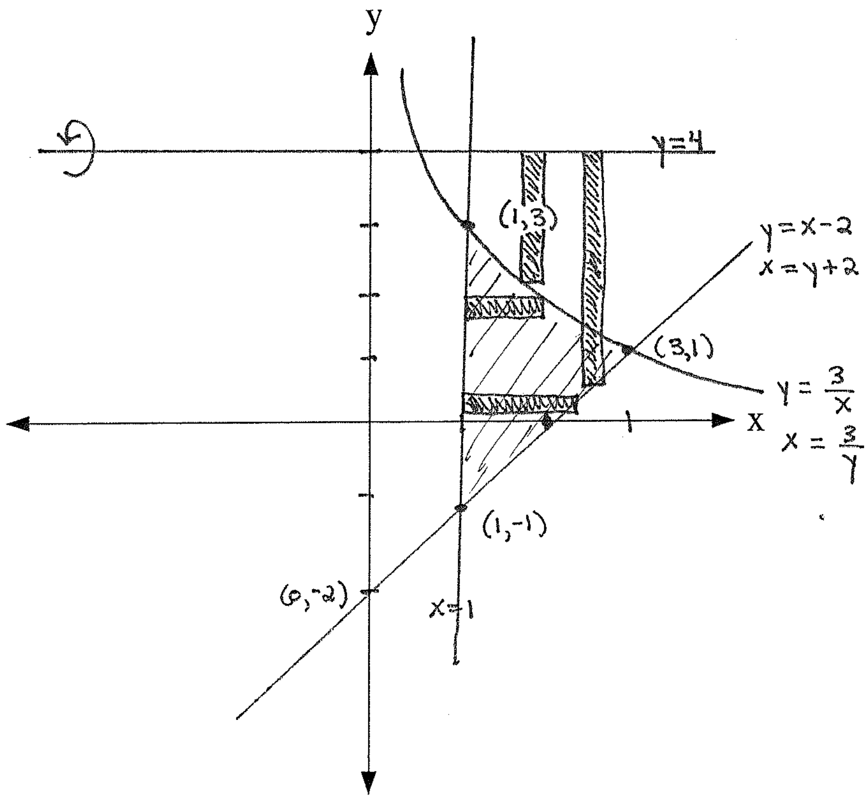
$$V = \int_0^2 \left(\pi 4^2 - \pi (2y)^2\right) dy$$

To find the volume of the resulting solid using the shell method, we would consider shells with average radius x and height $\frac{x}{2}$ as x varies from zero to four, which gives us:

$$V = \int_0^4 2\pi x \left(\frac{x}{2}\right) dx.$$

Hence these two integrals are equal.

3. Sketch the region bounded by $y = \frac{3}{x}$, $y = x - 2$ and $x = 1$. Set up the integrals used to find the volume obtained by revolving the region about $y = 4$ using (a) the disk method AND (b) the shell method. **DO NOT EVALUATE THE INTEGRALS!.**



$$\frac{3}{x} = x-2$$
$$3 = x^2 - 2x$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3, -1$$
$$(3,1) \ \& \ (-1,-3)$$

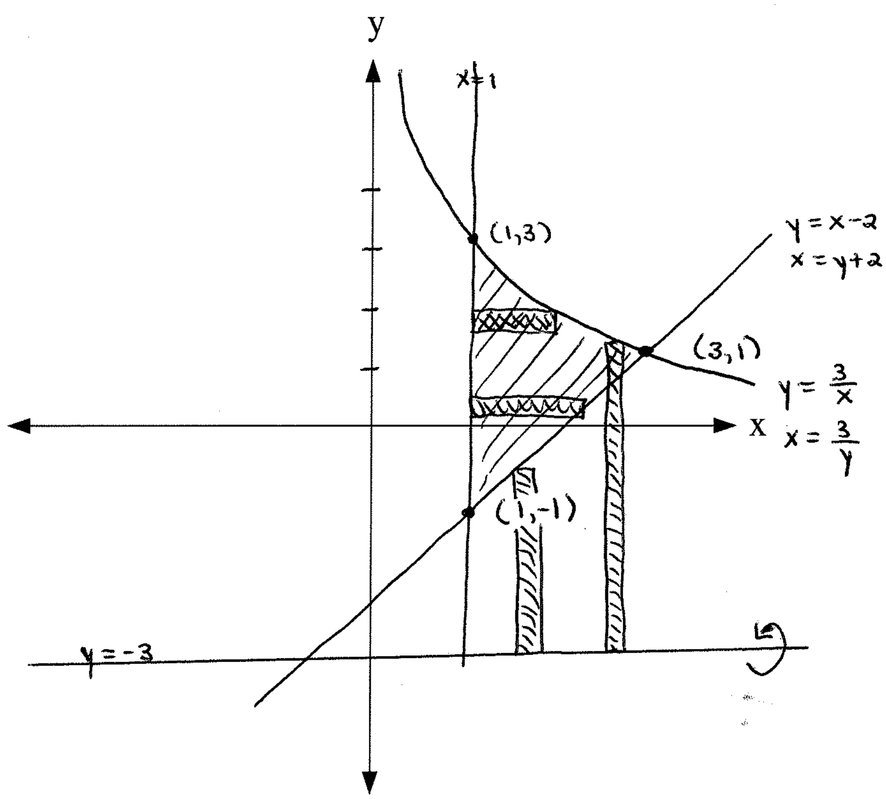
a) Disk Method

$$V = \int_1^3 \pi \left[(4-(x-2))^2 - \left(4 - \frac{3}{x}\right)^2 \right] dx$$

b) Shell Method

$$V = \int_{-1}^{1} 2\pi (4-y)((y+2)-1)\,dy + \int_1^3 2\pi(4-y)\left(\frac{3}{y} - 1\right) dy$$

4. Repeat question three, except rotate the region about $y = -3$.



a) Disk Method

$$V = \int_1^3 \pi \left[\left(\frac{3}{x} - (-3) \right)^2 - \left((x-2) - (-3) \right)^2 \right] dx$$

b) Shell Method

$$V = \int_{-1}^{1} 2\pi (y+3) \left((y+2) - 1 \right) dy + \int_1^3 2\pi (y+3) \left(\frac{3}{y} - 1 \right) dy$$

5. Find the length of the curve $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$ on the interval $[1, 3]$.

$$y = \tfrac{1}{4}x^4 + \tfrac{1}{8}x^{-2}$$

$$y' = x^3 - \tfrac{1}{4}x^{-3} = x^3 - \frac{1}{4x^3}$$

$$L = \int_1^3 \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)^2}\, dx$$

$$= \int_1^3 \sqrt{1 + x^6 - \frac{1}{2} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^3 \sqrt{x^6 + \frac{1}{2} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^3 \sqrt{\left(x^3 + \frac{1}{4x^3}\right)^2}\, dx$$

$$= \int_1^3 \left(x^3 + \tfrac{1}{4}x^{-3}\right) dx$$

$$= \tfrac{1}{4}x^4 + \tfrac{1}{4} \cdot \tfrac{1}{-2} x^{-2} \Big]_1^3$$

$$= \left(\frac{81}{4} - \frac{1}{72}\right) - \left(\frac{1}{4} - \frac{1}{8}\right)$$

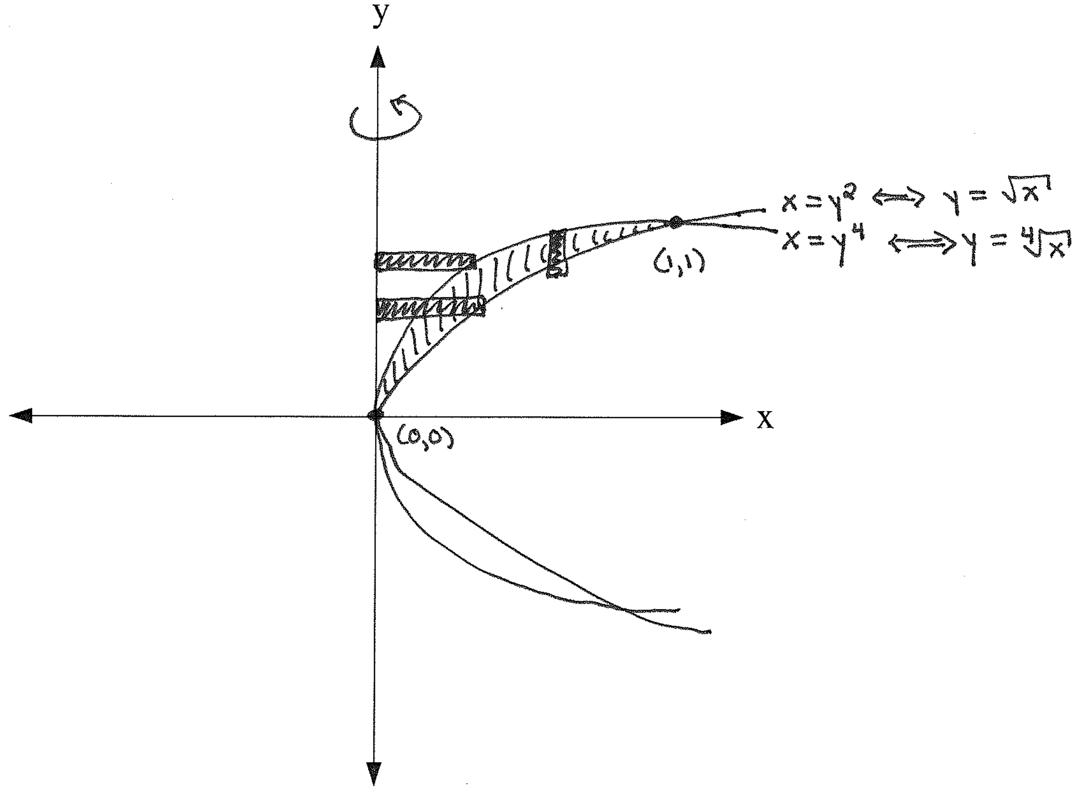
$$= \frac{80}{4} + \frac{8}{72}$$

$$= 20 + \frac{1}{9}$$

$$= \frac{181}{9}$$

6. The integral $\pi \int_0^1 (y^4 - y^8)dy$ represents the volume of a solid found using the disk method.

(a) Describe the solid. Use both a sketch and full sentences for a complete argument.



The solid is formed by rotating the region bound by $y = \sqrt{x}$ and $y = \sqrt[4]{x}$ about the y-axis.

$$V = \pi \int_0^1 [(y^2)^2 - (y^4)^2]\,dy$$

(b) Set up the integral to find the volume of the same solid using the shell method. **Do not evaluate the integrals!**

$$V = \int_0^1 2\pi x\,(\sqrt[4]{x} - \sqrt{x})\,dx$$

SOME QUESTIONS RELATED TO u-SUBSTITUTION!

7. Evaluate $\int_0^1 x\sqrt{1-x^4}dx$ by making a substitution and interpreting the resulting integral in terms of an area. Be sure to show your work and explain how you are interpreting in terms of an area.

Let $u = x^2$ if $x=0$, $u=0$

$du = 2x\,dx$ if $x=1$, $u=1$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\underbrace{\int_0^1 \sqrt{1-u^2}\,du}_{\frac{1}{4}\text{ circle centered at }(0,0)\text{ with radius }1} = \frac{1}{2}\left(\frac{1}{4}\pi(1)^2\right) = \frac{\pi}{8}$$

8. If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.

Let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

if $x=0$, $u=0$

if $x=3$, $u=9$

$$\Rightarrow \int_0^3 x f(x^2)dx = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$$

9. Find the volume of the solid obtained by revolving the region bounded by the curve $y = \sqrt{x} + 2$, the line tangent to this curve at $x = 1$, and $x = 0$ about the line $y = 1$.

Answer $= \frac{\pi}{4}$

Hint: start by finding the slope of the tangent line at $x = 1$,
then set up the equation of the line