WEEK 9 LAB

MATH 131 Section 2
October 25, 2018
Covering Sections 7.1-7.3

Your Name (Print): ANSWER KEY

Evaluate the following integrals:

1. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}}\, dx$ (Leave your final answer exact - with roots!)

$$= \int_{\frac{\pi}{6}}^{\pi/2} \frac{\cos^2 x}{\sqrt{\sin x}} \cdot \cos x \, dx$$

$$= \int_{\frac{\pi}{6}}^{\pi/2} \frac{(1 - \sin^2 x)}{\sqrt{\sin x}} \cos x \, dx$$

$u = \sin x$
$du = \cos x \, dx$
$x = \frac{\pi}{6} \Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$
$x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = \underline{1}$

$$= \int_{1/2}^{1} \frac{(1 - u^2)}{u^{1/2}} \, du$$

$$= \int_{1/2}^{1} \left(u^{-1/2} - u^{3/2}\right) du$$

$$= 2u^{1/2} - \frac{2}{5} u^{5/2} \Big]_{1/2}^{1}$$

$$= \left(2 - \frac{2}{5}\right) - \left(2\left(\frac{1}{2}\right)^{1/2} - \frac{2}{5}\left(\frac{1}{2}\right)^{5/2}\right)$$

$$= 2 - \frac{2}{5} - \frac{2}{\sqrt{2}} + \frac{2}{5 \cdot 4\sqrt{2}}$$

$$= \frac{8}{5} - \frac{2}{\sqrt{2}} + \frac{1}{10\sqrt{2}}$$

$$= \frac{16\sqrt{2} - 20 + 1}{10\sqrt{2}}$$

$$= \frac{16\sqrt{2} - 19}{10\sqrt{2}}$$

2. $\int_1^2 \frac{\ln x}{x^3}\,dx = -\frac{1}{2x^2}\ln x\Big]_1^2 + \frac{1}{2}\int_1^2 \frac{1}{x^3}\,dx$

$u = \ln x \qquad dv = x^{-3}dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^{-2}}{-2}$

$$= -\frac{\ln x}{2x^2} + \frac{1}{2}\cdot\frac{x^{-2}}{-2}\Big]_1^2$$

$$= \left(-\frac{\ln 2}{2(2)^2} + \frac{-1}{4(2)^2}\right) - \left(-\frac{\cancel{\ln 1}^{\,0}}{2(1)^2} + \frac{-1}{4(1)^2}\right)$$

$$= -\frac{1}{8}\ln 2 - \frac{1}{16} + \frac{1}{4} = -\frac{1}{8}\ln 2 - \frac{1}{16} + \frac{4}{16}$$

$$= -\frac{1}{8}\ln 2 + \frac{3}{16}$$

3. $\int_1^2 \frac{(\ln x)^3}{x}\,dx = \int_0^{\ln 2} u^3\,du = \frac{1}{4}u^4\Big]_0^{\ln 2} = \frac{1}{4}\left[(\ln 2)^4 - 0\right]$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \frac{(\ln 2)^4}{4}$$

if $x = 1$, then $u = \ln 1 = 0$

if $x = 2$, then $u = \ln 2$

4. $\int \tan^5 x\sqrt{\sec x}\,dx$ (Here is a situation where you might not be able to peel off exactly what you want...but you can make it so by multiplying by a fancy one!)

$$= \int \tan^4 x \sqrt{\sec x} \cdot \tan x \cdot \frac{\sec x}{\sec x}\,dx$$

$$= \int (\tan^2 x)^2 \cdot \frac{\sqrt{\sec x}}{\sec x} \cdot \sec x \tan x\,dx$$

$$= \int (\sec^2 x - 1)^2 (\sec x)^{-1/2} \cdot \sec x \tan x\,dx \qquad \begin{aligned} u &= \sec x \\ du &= \sec x \tan x\,dx \end{aligned}$$

$$= \int (u^2-1)^2 u^{-1/2}\,du$$

$$= \int (u^4 - 2u^2 + 1)\, u^{-1/2}\,du = \int \left(u^{7/2} - 2u^{3/2} + u^{-1/2}\right) du$$

$$= \frac{2}{9} u^{9/2} - 2\cdot\frac{2}{5} u^{5/2} + 2u^{1/2} + C$$

$$= \frac{2}{9} (\sec x)^{9/2} - \frac{4}{5} (\sec x)^{5/2} + 2\sqrt{\sec x} + C$$

5. $\int_4^{16} \frac{1}{1-\sqrt{x}}\,dx$ (Don't be afraid to experiment!)

$$\begin{aligned} u &= 1 - \sqrt{x} \\ \sqrt{x} &= 1 - u \\ x &= (1-u)^2 \\ dx &= -2(1-u)\,du \end{aligned} \qquad \begin{aligned} &\text{if } x=4, \quad u = 1-\sqrt{4} = 1-2 = -1 \\ &\text{if } x=16, \quad u = 1-\sqrt{16} = 1-4 = -3 \end{aligned}$$

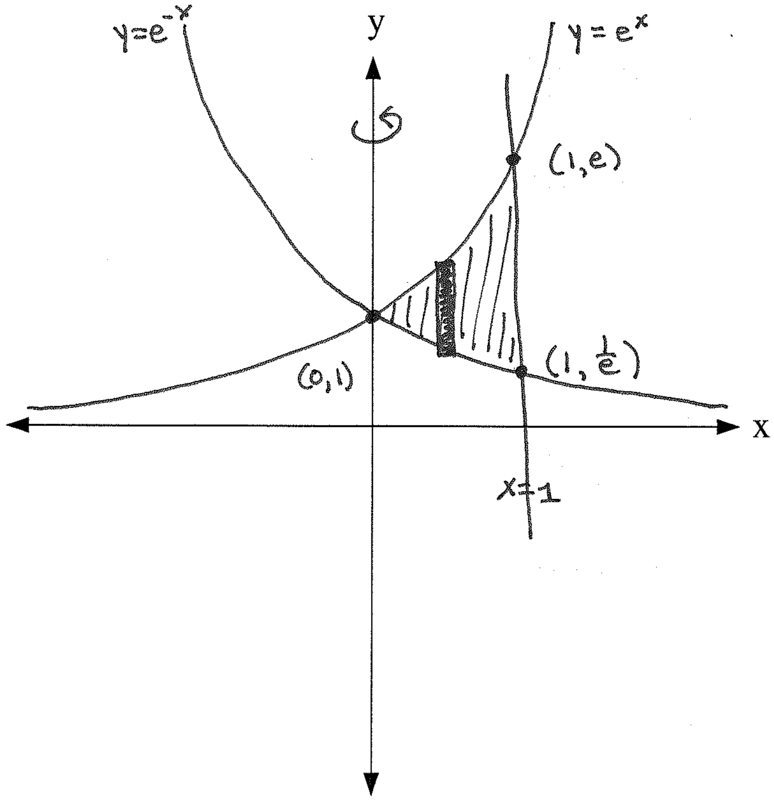
$$\int_4^{16} \frac{1}{1-\sqrt{x}}\,dx = \int_{-1}^{-3} \frac{1}{u}\left(-2(1-u)\right) du = \int_{-1}^{-3} \frac{2u-2}{u}\,du = \int_{-1}^{-3} \left(2 - 2u^{-1}\right) du$$

$$= 2u - 2\ln|u| \Big]_{-1}^{-3} = \left(2(-3) - 2\ln|-3|\right) - \left(2(-1) - 2\ln|-1|\right)$$

$$= -6 - 2\ln 3 + 2 + \cancel{2\ln 1}^{\,0}$$

$$= -4 - 2\ln 3$$

6. Find the volume generated by rotating the region bounded by $y = e^x$, $y = e^{-x}$, and $x = 1$ about the y-axis. Include a neatly drawn graph with the appropriate functions and important points labeled clearly and the region shaded.



+6

$$V = \int_0^1 2\pi x\,(e^x - e^{-x})\,dx = 2\pi \int_0^1 xe^x\,dx - 2\pi \int_0^1 xe^{-x}\,dx$$

$u = x \quad dv = e^x dx \qquad u = x \quad dv = e^{-x} dx$

$du = dx \quad v = e^x \qquad du = dx \quad v = -e^{-x}$

$$= 2\pi \left\{ \left[\left. xe^x \right]_0^1 - \int_0^1 e^x\,dx \right] - \left[\left. -xe^{-x} \right]_0^1 + \int_0^1 e^{-x}\,dx \right] \right\}$$

$$= 2\pi \left\{ \left[xe^x - e^x \right]_0^1 - \left[-xe^{-x} - e^{-x} \right]_0^1 \right\}$$

$$= 2\pi \left\{ \left. xe^x - e^x + xe^{-x} + e^{-x} \right]_0^1 \right\}$$

$$= 2\pi \left\{ (\cancel{e} - \cancel{e} + e^{-1} + e^{-1}) - (0 - \cancel{1} + 0 + \cancel{1}) \right\}$$

$$= 2\pi \left[\frac{2}{e} \right] = \boxed{\frac{4\pi}{e}}$$

7. $\int \sin(\ln x)\,dx$

$$u = \sin(\ln x) \qquad dv = dx$$
$$du = \frac{\cos(\ln x)}{x}\,dx \qquad v = x$$

$$\int \sin(\ln x)dx = x\sin(\ln x) - \int \cancel{x} \cdot \frac{\cos(\ln x)}{\cancel{x}}\,dx$$

$$u = \cos(\ln x) \qquad dv = dx$$
$$du = \frac{-\sin(\ln x)}{x}\,dx \qquad v = x$$

$$\int \sin(\ln x)dx = x\sin(\ln x) - \left[x\cos(\ln x) + \int x \cdot \frac{+\sin(\ln x)}{x}\,dx\right.$$

$$\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)dx$$

$$2\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x)$$

$$\int \sin(\ln x)dx = \frac{1}{2}\left[x\sin(\ln x) - x\cos(\ln x)\right] + C$$

8. $\int_0^{\frac{\pi}{4}} \theta \sec\theta \tan\theta \, d\theta$

ANS: $\dfrac{\pi\sqrt{2}}{4} - \ln|\sqrt{2} + 1|$

Hint: use integration by parts!

9. Find the area of the region bound by $y = \arctan x$, $y = 0$ and $x = \sqrt{3}$.

$\boxed{\frac{\pi\sqrt{3}}{3} - \ln 2}$

10. Evaluate $\int x^5 e^{x^2}\,dx$ by first making a substitution and then using integration by parts.

$\boxed{\frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + C}$

11. Evaluate $\int_0^1 x5^x\,dx$.

$\boxed{\frac{5\ln 5 - 4}{(\ln 5)^2}}$