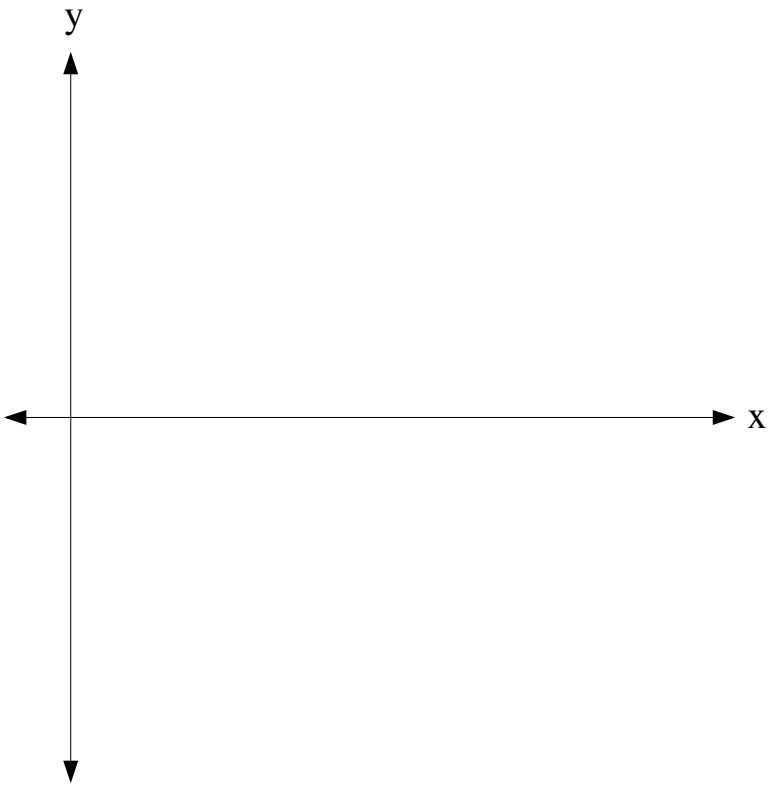
Finding Surface Area

MATH 131: Calculus II, Sections 2 and 3

What happens when we take a curve $y = f(x)$ and rotate it about a line? We form a surface of revolution, like the outer shell of a solid of revolution. What is the area of this surface? Let's figure it out!

Consider the following curve $y = f(x)$ on the interval $[a, b]$.



Our first step is, as usual, to _____.

Then, as in Section 6.5, we estimate the curve with _____.

Illustrate the two steps above on the graph.

Next we rotate the _____ about the x-axis to form bands. These bands look like part of a cone and are called frustums.

The surface area of a frustum = 2π(average radius)(slant height)= $2\pi\left(\frac{r_1 + r_2}{2}\right)l$.

Ok, so if we have the interval $[x_{i-1}, x_i]$, the average radius of the frustum on that interval is _____.

Note that this is a bit confusing because we are considering distinct x values. So let's use another old friend, the Intermediate Value Theorem!

By the Intermediate Value Theorem, there exists _____ in _____ such that

_____.

The slant height is equal to the _____ which is equal to _____.

Thus adding all the frustum surface areas together we get _____

Giving us that the surface area is $SA =$ _____