Groupwork: Equal Sets and Power Sets

MATH 135: First Steps into Advanced Mathematics
February 7, 2017 Name (Print): ______________________

PART I:

(1) **Review of Key Idea:** If A is a set, what does it mean if $X \in \mathscr{P}(A)$? Why am I using X and not x?

(2) Let $X = \{\{1, 2, 3\}, \{4, 5\}, 6\}$.

(a) How many elements does X have?

(b) What is $\mathscr{P}(X)$?

(c) List a subset of X of size two. What can you say about this subset?

(d) Decide whether each of the following is true or false. If it is false, explain why in a sentence.

(i) $\{4, 5\} \in X$

(ii) $4 \in X$

(iii) $\{4, 5\} \subseteq X$

(iv) $\{6\} \subseteq X$

(v) $\{6\} \in \mathscr{P}(X)$

(vi) $\{6\} \subseteq \mathscr{P}(X)$

(vii) $\{\{4, 5\}\} \subseteq X$

(viii) $\{\{4, 5\}\} \in \mathscr{P}(X)$

(ix) $\{4, 5\} \in \mathscr{P}(X)$

(x) $\{\emptyset\} \subseteq \mathscr{P}(X)$

PART II:

Let A and B be sets. Consider DeMorgan's law of sets that says $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.

(1) Suppose that we want to prove this with an element argument. What will we need to show?

(2) WARM UP: Negate the following statement: The number x is even or the function f is continuous. (Can you see why this is a warm up for the next question?)

(3) Hopefully in (1) you noted that we had TWO things to show. These should appear as separate paragraphs in your proof. Then you should have a concluding statement, which combines the two results. The conclusion should be in its own paragraph. Thus these types of proofs will have at least three paragraphs (often you also have a fourth paragraph at the very beginning of the proof if you have some assumptions that apply to both cases). Now write the proof! (Don't forget your definitions worksheet!)

PART III:

Now let's look at power sets with set equality!

(1) Prove the following theorem: **Theorem**: Let A and B be sets. Then $A \subseteq B$ iff $\mathscr{P}(A) \subseteq \mathscr{P}(B)$.

(2) Prove the following theorem: **Theorem**: Let A and B be sets. Then $\mathscr{P}(A \cap B) = \mathscr{P}(A) \cap \mathscr{P}(B)$.

(3) This problem has three parts:

(a) Prove the following theorem: **Theorem**: Let A and B be sets. Then $\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$.

(b) Provide a counterexample to show that it is not necessarily true that $\mathscr{P}(A) \cup \mathscr{P}(B) = \mathscr{P}(A \cup B)$.

(c) Is it *ever* true that $\mathscr{P}(A) \cup \mathscr{P}(B) = \mathscr{P}(A \cup B)$?