## MATH 135 HOMEWORK - Week 5

MATH 135: First Steps into Advanced Mathematics
Due February 17, 2017 at 2:00pm
Name (Print): $\qquad$
Remember that you may NOT discuss this assignment with others!
Consider the theorem: For all integers $x$, if 4 does not divide $x^{2}$, then $x$ is odd.
(Note that your answers below should be specifically in relation to this theorem not in relation to statements in general. In other words, if you want to say "assume the hypothesis is true" you actually want to tell me what the hypothesis is. So you should say "assume BLAH" where BLAH comes directly from the statement above.)
(a) State your assumptions and final conclusion if this statement by direct proof.
(b) State your assumptions and final conclusion if proving this statement by proof by contraposition.
(c) State your assumptions and final conclusion if proving this statement by proof by contradiction.
(d) Write any definitions you need to prove the theorem.
(e) Do at least two examples that illustrate the concepts in the statement. In particular, produce one $x$ which fulfills the hypothesis of the statement and one that does not. Be sure to explain what your examples are showing.
(f) Write a rough draft/outline of a proof of the theorem. Which method did you choose?
(g) Write a final draft of the proof you worked on in part (g). It should be beautiful with nothing crossed out, etc.
(h) Suppose the original statement was "For all integers $x, 4$ does not divide $x^{2}$ if and only if $x$ is odd." What additional work would we need to do to complete the proof? What method of proof would you use to do this? Explain why very carefully.

