Homework Week 6

MATH 204: Linear Algebra   Due October 5, 2018 **by 1:55pm**

   Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate. Complete these exercises on a separate paper.

1. Number 6 from Section 1.8, page 69. Read the directions carefully! They ask you to answer two questions!

2. (a) Carefully prove the following theorems. Be sure to refer to any theorems or facts very specifically.
   Theorem : Assume that $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with standard $m \times n$ matrix A. If T is one-to-one, then $m \geq n$.

   (b) Show that the converse of the theorem is not true. (Note that you will need to state what the converse is in order to do this! If you don't remember what the converse is, check out Hammack's book. You will find the converse defined in the Logic chapter, page 44 of the text.)

3. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation. Can there be a vector $\mathbf{b}$ in $\mathbb{R}^m$ such that T sends *exactly* two vectors to $\mathbf{b}$? Justify your answer carefully with appropriate theorem(s).

4. Number 10 from Section 1.8, page 69.

5. Number 24 from Section 1.8, page 70. Hint: If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span $\mathbb{R}^n$, how can we rewrite the vector $\mathbf{x}$?

6. Consider the transformations defined in Exercises 17, 19, and 20 of Section 1.9. For each one, show that it is not one-to-one by giving two input vectors that have the same image. You should be able to find the vectors "by inspection". However your answer should include a calculation that you are verifying that your vectors "work."