Homework Week 7

MATH 204: Linear Algebra
Due October 20, 2017 **by 1:55pm** Name (Print): ____________________

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. see website

2. see website

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be an arbitrary 2×2 matrix and r be a scalar. Prove $\det(rA) = r^2 \det(A)$.

4. One of these things is not like the others: In three of the following, it is impossible to give an example that meets the stated criteria, while in the fourth, an example is possible. Give an example in the one case where it is possible to do so, and PROVE that your example fits the bill. For the remaining three cases, PROVE that it is impossible to give an example. Your proofs should be short but rigorous. Make sure you cite theorems to justify your claims; if you use the Invertible Matrix Theorem, state explicitly which parts you are using as follows. Example: "Since A^T is invertible, by the Invertible Matrix Theorem, the columns of A are independent."

(a) A linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ that is one-to-one but not onto $\mathbb{R}^5$.

(b) A 5×5 invertible matrix A whose transpose is the product of elementary matrices.

(c) A 5×5 matrix A with 5 pivots such that A's first column is a linear combination of A's second and third columns.

(d) An onto linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ whose standard matrix has an all-zero column.

5. Suppose a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ has the property that $T(\vec{u}) = T(\vec{v})$ for some pair of distinct vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^n$. Can T map $\mathbb{R}^n$ onto $\mathbb{R}^n$? Why or why not?

6. see website

7. see website