

Homework Week 9

MATH 204: Linear Algebra

Due October 27, 2017 by 1:55pm

Name (Print): _____

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. Determine the values of k that make the matrix $A = \begin{bmatrix} k & k & 9 \\ 2 & 0 & 0 \\ 3 & 4 & k \end{bmatrix}$ singular. Justify your answer; show your work.

2. An $n \times n$ matrix A is said to be **conjugate** to the $n \times n$ matrix B , if there exists an invertible $n \times n$ matrix M so that $A = MBM^{-1}$. Prove: If A is conjugate to B , then $\det(A) = \det(B)$.

3. An $n \times n$ invertible matrix U is **orthogonal** if $U^{-1} = U^T$. Prove that if U is orthogonal, then $\det(U) = \pm 1$.

4. Let V be the union of the first and third quadrants in the xy -plane. That is, $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$.

(a) Suppose \vec{u} is in V and c is any real number. Is $c\vec{u}$ in V ? Why? Carefully justify your answer.

(b) What does this tell you about V ?

(c) Carefully show that V is NOT closed under vector addition.

(d) What does this tell you about V ?