Homework Week 11

MATH 204: Linear Algebra
Due November 10, 2017 **by 1:55pm** Name (Print): _____

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. Assume A is an $n \times n$ matrix. Prove: If Nul $A = \{\vec{0}\}$, then det $(A^T) \neq 0$.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$. Give explicit descriptions of Nul A and Col A, i.e. as spans of sets of vectors.

Generalizing from our previous definition, we have: **Definition:** A linear transformation $T : V \to W$ where V and W are vector spaces, is **one-to-one** if whenever $T(\vec{u}) = T(\vec{v})$, then $\vec{u} = \vec{v}$.

3. Let $T : \mathbb{R}^2 \to M_{22}$ be defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x & y \\ y & 2x \end{bmatrix}$.

   (a) Prove T is a linear transformation.

   (b) Determine whether T is one-to-one. Prove your answer. (Assume $T(\vec{u}) = T(\vec{v})$. Must $\vec{u} = \vec{v}$?)

4. Let $T : M_{22} \to M_{22}$ be defined by $T(A) = A + A^T$.

   (a) Prove T is a linear transformation.

   (b) Find a description of $\ker(T)$ in terms of the components of the matrices. That is determine how to write the kernel in the form

   $$\ker(T) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \text{some conditions on } a, b, c, d \right\}$$

   and justify your answer.

   (c) BONUS (1 point): Write $\ker(T)$ as the span of a set of vectors (matrices) in M_{22}.