Homework Week 12

MATH 204: Linear Algebra
Due November 17, 2017 **by 1:55pm** Name (Print): ________________________

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. Find a basis for the subspace of M_{22} spanned by the matrices

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-2 & 0\\0 & 2\end{bmatrix}, \begin{bmatrix}3 & -1\\1 & -1\end{bmatrix}, \begin{bmatrix}5 & -3\\3 & -4\end{bmatrix}, \begin{bmatrix}2 & -1\\1 & 0\end{bmatrix}.$$

(Hint: Since we add and scalar multiply matrices just like we do with vectors in $\mathbb{R}^n$, we can think of matrices in M_{22} as vectors in $\mathbb{R}^4$ by listing all of the matrix entries vertically in a column. Now you can solve the problem in $\mathbb{R}^4$ and convert the answer back to matrices! Cool!)

2. Let $A = \begin{bmatrix}1 & 2 & 1 & 1 & 1\\3 & 6 & 4 & 3 & 1\\4 & 8 & 5 & 5 & 5\\7 & 12 & 7 & 7 & 7\end{bmatrix}$.

(a) Find a basis for Nul A.

(b) Find a basis for Col A.

3. Recall that $\mathbb{P}_2$ is the set of all polynomials of degree less than or equal to two. Define a linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\vec{p}) = \begin{bmatrix}\vec{p}(0)\\\vec{p}(0)\end{bmatrix}$.

(a) Determine the kernel of T. Write it explicitly as a spanning set.

(b) Determine the range of T. Is T onto?

4. State clearly whether each statement is true or false, then justify your answer.

(a) The set of all polynomials $\vec{p}$ in $\mathbb{P}_3$ satisfying $\vec{p}(0) = 1$ is a subspace of $\mathbb{P}_3$.

(b) If $A = \begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}$, then Col $A = \mathbb{R}^3$.

(c) The matrices $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$, $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ and $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$ form a linearly independent set in M_{22}. (Hint: Use the hint from exercise 1!)

5. Prove the following Theorem: Suppose V and W are vector spaces and $T : V \to W$ is a one-to-one linear transformation. If $\{T(\vec{u}_1), T(\vec{u}_2), \ldots, T(\vec{u}_n)\}$ is linearly dependent, then $\{\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n\}$ is linearly dependent. (This theorem together with Theorem 8 on page 221 tells us why the procedure in Example 6 works! Hint for the proof: Make sure you use your hypotheses!)