

Homework Week 10

MATH 204: Linear Algebra

Due November 3, 2017 by 1:55pm

Name (Print): _____

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. This example will help you understand that vector spaces depend on both the set and how the operations of addition and scalar multiplication are defined. Let $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$, the set of positive real numbers. Since the operations are not the usual ones, we will use the symbols discussed in class. Define “addition” as multiplication: $x \oplus y = xy$ and “scalar multiplication” as raising to a power: $c \odot x = x^c$. Using these operations write full proofs of the following:

(a) Verify Axiom 1 (Here your proof should begin with “Let $x, y \in \mathbb{R}^+$ ” ...).

(b) Verify Axiom 2.

(c) Verify Axiom 4.

(d) Verify Axiom 6.

(e) Verify Axiom 8 (Note that this looks like: $(c + d) \odot x = (c \odot x) \oplus (d \odot x)$).

(I am only asking you to show me proofs that five of the axioms hold. You should check the rest as well to be sure you believe this is a vector space!)

2. Let H be the set of all singular 2×2 matrices. Is H a subspace of M_{22} ? Carefully justify your answer with a proof or counterexample.

3. Let K be the set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$ where $a + b = 0$. Is K a subspace of M_{22} ? Carefully justify your answer with a proof or counterexample.

4. Let $W = \left\{ \begin{bmatrix} 2s + 4t \\ 2s \\ 2s - 3t \\ 5t \end{bmatrix} \in \mathbb{R}^4 : s, t \in \mathbb{R} \right\}$. (a) Give an explicit example of a non-zero vector $\vec{v} \in W$. (b) Is

W a subspace of \mathbb{R}^4 ? Justify your answer very carefully with a proof (Hint: If it IS a subspace, feel free to use a very nice theorem!).

5. Let $J = \left\{ \begin{bmatrix} 4a + 3b \\ a + 3b + c \\ 1 \\ 3b - 2c \end{bmatrix} \in \mathbb{R}^4 : s, t \in \mathbb{R} \right\}$. (a) Give an explicit example of a non-zero vector $\vec{w} \in J$. (b)

Is J a subspace of \mathbb{R}^4 ? Justify your answer very carefully with a proof (Hint: If it IS a subspace, feel free to use a very nice theorem!).