Homework Week 14

MATH 204: Linear Algebra
Due December 1, 2017 **by 1:55pm** Name (Print): ______________________

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{P}_2$ defined by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = (a+b) + at + (b-a)t^2$. Show that T is one-to-one. (Hint: One method is to determine $\ker T$. Why?)

2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation that is one-to-one. What is the dimension of the Range of T? (Hint: This linear transformation is a matrix transformation!) Justify with a proof.

3. What is the dimension of $\mathbb{P}_3$? Consider the following polynomials: 1, $1-t$, $2-4t+t^2$ and $6-18t+9t^2-t^3$. Using Theorem 4.4 (and other appropriate theorems and definitions), prove that these polynomials form a basis of $\mathbb{P}_3$.

4. Find the dimension of the subspace spanned by $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -8 \\ 6 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$. Justify your conclusion carefully.

5. Complete Section 4.6 number 4 on page 238 in our text.

6. Suppose a nonhomogeneous system of seven linear equations in ten unknowns has a solution with three free variables. Is it possible to change some constants on the equations' right sides (i.e. to some coefficients in the solution vector) to make the new system inconsistent? Why or why not?

7. Suppose you are given a homogeneous system of nine linear equations in eleven variables. Is it possible that all solutions of the system are multiples of one fixed nonzero solution? Why or why not?