# Homework Week 4

MATH 204: Linear Algebra
Due September 22, 2017 **by 1:55pm** Name (Print): ____________________

Remember that although you may discuss this assignment with others, your write up should be your own. **Do not share your write-up, look at other's write-ups, discuss word for word how something should be proved, etc.** Be sure to note with whom you collaborate if you do collaborate.

1. Number 18 from Section 1.4, page 41. The instructions and the matrix $B$ are located above Exercise 17. Justify your answer using an appropriate theorem.

2. This question is designed to make you think about pivot positions in the rows and/or columns of a (coefficient) matrix $A$.

(a) Suppose $A$ is a $4 \times 4$ matrix and $b \in \mathbb{R}^4$ is a vector such that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Does the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ have a solution? If so, is the solution unique? Prove your answer very clearly, justifying your assertions very carefully.

(b) Suppose $A$ is a $4 \times 3$ matrix and $b \in \mathbb{R}^4$ is a vector such that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Does the equation $A\mathbf{x} = \mathbf{c}$ have a solution for all $c \in \mathbb{R}^4$? Prove your answer very clearly, justifying your assertions very carefully. Use an appropriate theorem.

3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 9 & 15 \\ 2 & 5 & h \end{bmatrix}$. For what values of $h$ do the columns of $A$ span $\mathbb{R}^3$? Be sure to show your work and justify your answer with an appropriate theorem.

4. Practice with Proof-writing: Prove part (iv) of the Algebraic Properties of Vectors in $R^n$ Theorem (p. 27). See your class notes and the solution to Practice Problem 1 of Section 1.3 for an example of how such a proof should go.

5. Number 8 from Section 1.5, page 48. Be careful. You are given the coefficient matrix, not the augmented matrix for homogeneous system $A\mathbf{x} = \mathbf{0}$.

Note this has a second page! Keep going!

6. (a) Describe all solutions of the homogeneous system below in parametric vector form. What type of geometric set does it form in $\mathbb{R}^4$?

$$\begin{aligned} x_1 + 3x_2 - 3x_3 + 7x_4 &= 0 \\ x_2 - 4x_3 + 5x_4 &= 0 \\ -2x_1 - 8x_2 + 14x_3 - 24x_4 &= 0 \end{aligned}$$

(b) Describe all solutions of the non-homogeneous system below in parametric vector form. Describe the geometry of the solution set compared to the solution set of part (a).

$$\begin{aligned} x_1 + 3x_2 - 3x_3 + 7x_4 &= 1 \\ x_2 - 4x_3 + 5x_4 &= 2 \\ -2x_1 - 8x_2 + 14x_3 - 24x_4 &= -6 \end{aligned}$$

7. Answer each of the following. Give clear, careful, short proofs that your answers are correct, using theorems and facts.

(a) Suppose $A$ is a $5 \times 5$ matrix with 4 pivot positions.

(i) Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

(ii) Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^5$?

(b) Suppose $A$ is a $5 \times 4$ matrix with 4 pivot positions.

(i) Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

(ii) Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^5$?

(c) Suppose $A$ is a $4 \times 5$ matrix with 4 pivot positions.

(i) Must the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

(ii) Must the equation $A\mathbf{x} = \mathbf{b}$ have a solution for EVERY $\mathbf{b} \in \mathbb{R}^4$?

8. Number 6 from Section 1.7, page 61.