The Final and Other Business

MATH 204: Linear Algebra

Review Session: We will have a review session on Monday, December 11th from 11:00am until Noon in Napier 102. Attendance is optional and you are welcome to come for any portion of the review session. Bring questions.

Office Hours: I will hold office hours Monday, December 11th from 4:00pm until 5:00pm. If you have conflicts at that time and still wish to see me, **please** make an appointment.

The Where and When of the Final: The final exam is on Tuesday, December 12th from 8:30am until 11:30am in Stern 303. The Final Exam is worth 24% of your final exam grade.

NOTE: Similar to the midterm exams, there will be short answer questions in addition to problems. For example, I could give you a few statements and ask you to determine whether each was true or false and to prove or give a counterexample for each. Similarly, I could ask you to give me an example of something or justify that no such example exists.

NOTE: The exam is cumulative and will be over all the material covered in Sections 1.1-1.5, 1.7-1.9, 2.1-2.3, 3.1, 3.2, 4.1-4.6, and 5.1-5.3. This is a **rough** guideline. You should be sure to review your homework, group work, quizzes and notes from these sections. Also take a look at your midterm review sheets!

NEW (since Exam 3) Warnings, Rules, Facts and Theorems: You should know and be able to use the following additional theorems and facts. Hopefully you already have these in your notes and/or on flashcards! These are in addition to the theorems and facts we covered for the midterms!

1. More Vectors than in a Basis Theorem (Theorem 4.9, page 227)
2. Bases Have the Same Size Theorem (Theorem 4.10, page 228)
3. Expansion Theorem (Theorem 4.11, page 229)
4. The Basis Theorem (Theorem 4.12, page 229)
5. Dimension of Nul A and Col A (Facts, page 230)
6. Row Equivalent Matrices and Their Row Spaces (Theorem 13, page 233)
7. The Rank Theorem (Theorem 4.14, page 235)
8. Extension of the Invertible Matrix Theorem (Theorem, page 237)
9. Eigenvalues of Triangular Matrices (Theorem 5.1, page 271)
10. Eigenvalues Corresponding to Distinct Eigenvalues (Theorem 5.2, page 272)
11. Extension II of the Invertible Matrix Theorem (Theorem, page 277)
12. Eigenvalues and the Characteristic Equation (Fact, page 278)
13. Eigenvalues of Similar Matrices (Theorem 5.4, page 279)
14. Warnings about Similar Matrices (Warning, page 279)
15. The Diagonalization Theorem (Theorem 5.5, page 284)
16. Diagonalizability and Distinct Eigenvalues (Theorem 5.6, page 286)
17. Dimension of Eigenspaces Theorem (Theorem 5.7, page 287)

NEW (since Exam 3) Definitions: You have been working hard on definitions! Be sure you have memorized these terms, in addition to all the definitions you learned for the midterms, for the final exam: dimension, finite-dimensional, infinite-dimensional, row space, rank, eigenvector, eigenvalue, eigenspace, characteristic equation, characteristic polynomials, similar matrices, diagonalizable, eigenvector basis. You should know how to use these as well as have a good definition of them memorized.

Be sure to...

(1) review your definitions and theorems.

(2) practice finding examples that satisfy or do not satisfy particular requirements

(3) practice problems with**out** your book or notes or collaborators. (If you haven't done all the practice problems I assigned, go back and work through those. There are some really great questions!)

(4) bring a pencil (or several!) with a good eraser.

(5) ask me questions if you are stuck or need clarification.

(6) breathe!

Some Practice Exercises

1. Let $\mathbb{F}$ denote the set of all functions that are defined for all real numbers.

   (a) Let $\mathbb{W} = \{f \in \mathbb{F} | f(1) = 4f(2)\}$. Is $\mathbb{W}$ a subspace of $\mathbb{F}$?

   (b) Let $\mathbb{W} = \{f \in \mathbb{F} | f(1) + f(2) = 1\}$. Is $\mathbb{W}$ a subspace of $\mathbb{F}$?

2. Consider the following system of equations:

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + hx_2 &= k \end{aligned}$$

   Find the values of h and k so that the system has:

   (a) No solution

   (b) A unique solution

   (c) Infinitely many solutions

3. Determine whether or not each of the following is possible. If possible, given an example. Justify each answer.

   (a) A system of four linear equations and four unknowns with exactly four solutions.

   (b) A system of three linear equations and four unknowns that does not have a solution.

   (c) A nonzero 3×3 matrix that has no inverse.

   (d) An invertible matrix A such that $\det A^T = 0$.

   (e) A square matrix A such that $AAA = I$ and $\det A = 2$.

   (f) Invertible matrices A and B such that $\det(A + B) = 0$.

   (g) A 2×2 matrix other than the identity matrix which is its own inverse.

   (h) A linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$ which is not one-to-one.

4. Prove: If A and B are similar $n \times n$ matrices and A is not invertible, then B is not invertible.

THANK YOU FOR A GREAT CLASS!!!

GOOD LUCK!!! HAVE A GREAT WINTER BREAK!!! KEEP IN TOUCH!!!