

Introduction to Linear Transformations

MATH 204: Linear Algebra
Prepare for class September 26, 2018

Name (Print): _____

After reading Section 1.8, answer the following questions.

1. State the definition of a **transformation**.

2. What is the difference between the codomain and the range of a transformation?

3. Consider the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 3x_2 \end{bmatrix}$.

(a) What is the image under T of $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$?

(b) What matrix could represent this transformation T ?

(c) Find an $\mathbf{x} \in \mathbb{R}^2$ whose image under T is $\mathbf{b} = \begin{bmatrix} 4 \\ -6 \\ 15 \end{bmatrix}$.

(d) Determine if $\mathbf{c} = \begin{bmatrix} 5 \\ 2 \\ 12 \end{bmatrix}$ is in the range of T .

4. State the definition of a **linear transformation**.

5. Use the definition above to prove the property that $T(\mathbf{0}) = \mathbf{0}$ if T is a linear transformation.

6. Similarly use the definition of linear transformation to prove the property that $T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$ if T is a linear transformation.

7. Is the transformation in question 3 linear? Justify.

8. Consider the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$. Is this transformation linear? Justify.