Eigenvectors and Eigenvalues, and the Characteristic Equation

MATH 204: Linear Algebra
Prepare for class December 5, 2018 (Happy December!) Name (Print): _____

After reading Section 5.1 and Section 5.2, work through the following ideas.

1. State the definition of the eigenspace of A corresponding to λ.

2. Let $B = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. One eigenvalue for B is $\lambda = 4$. Find a basis for the eigenspace of B corresponding to this λ.

3. State Theorem 5.1: Eigenvalues of Triangular Matrices.

4. Now let's prove Theorem 5.1 in the 3×3 case.

(a) Let A be a 3×3 upper triangular matrix. Write down it's general form.

(b) Now write what $A - \lambda I$ looks like.

(c) If λ is an eigenvalue of A, what must be true? When is this true for your matrix in (b)?

(d) What is your conclusion?

5. What fact can be added to the Invertible Matrix Theorem according to the text on page 272? Be sure to word it as if it could be listed as equivalent to the statements already in the Invertible Matrix Theorem.

6. State Theorem 5.2.

7. The technique the authors use to prove Theorem 2 is called proof by contradiction. Give an outline of this proof.

8. State the definition of the characteristic equation of a matrix A, as well as the definition of the characteristic polynomial.

9. State the FACT on page 278 about eigenvalues and the characteristic equation.

10. Try Exercise 1 on page 281.

11. The FACT on the top of page 277 should be familiar. We saw it back in Section 3.2! Write it here and include what each of the variables represents.

12. State the Extension of the Invertible Matrix Theorem on page 277.

13. What is the multiplicity of an eigenvalue?

14. Write down any questions you have on the reading.