Homework Assignment 10
Math 3001 — Fall 2014
Due Friday, October 31

Exercises:

4.4.2. Show that $f(x) = 1/x^2$ is uniformly continuous on the set $[1, \infty)$ but not on the set $(0, 1]$.

4.4.3. Show that if $f(x)$ is continuous on $[a, b]$ with $f(x) > 0$ for all $a \leq x \leq b$, then $1/f(x)$ is bounded on $[a, b]$.

4.4.9. A function $f\colon A \to \mathbf{R}$ is called *Lipschitz* if there exists a bound $M > 0$ such that

$$\left| \frac{f(x) - f(y)}{x - y} \right| \leq M$$

for all $x, y \in A$. Geometrically speaking, a function f is Lipschitz if there is a uniform bound on the magnitude of the slopes of lines drawn through any two points on the graph of f.

(a) Show that if $f\colon A \to \mathbf{R}$ is Lipschitz, then f is uniformly continuous on A.

4.5.5. Finish the proof of the Intermediate Value Theorem using the Axiom of Completeness started on the bottom of page 122 (*Proof.* **I.**)

4.5.7. Let f be a continuous function on the close interval $[0, 1]$ with range also contained in $[0, 1]$. Prove that f must have a fixed point; that is, show that $f(x) = x$ for at least one value of $x \in [0, 1]$.