Homework Assignment 4
Math 3001 — Fall 2014
Due Friday, September 19

Exercises:

2.2.1. Verify, using the definition of convergence of a sequence, that the following sequences converge to the proposed limit.

(a) $\lim \frac{1}{(6n^2+1)} = 0$.

(b) $\lim \frac{3n+1}{2n+5} = \frac{3}{2}$.

2.3.1. Show that the constant sequence $(a, a, a, a, \ldots)$ converges to a.

2.3.3. (Squeeze Theorem) Show that if $x_n \leq y_n \leq z_n$ for all $n \in \mathbf{N}$, and if $\lim x_n = \lim z_n = \ell$, then $\lim y_n = \ell$ as well.

2.3.4. Show that the limits, if they exist, must be unique. In other words, assume $\lim a_n = \ell_1$ and $\lim a_n = \ell_2$, then prove that $\ell_1 = \ell_2$.

2.3.6. (a) Show that if $(b_n) \to b$, then the sequence of absolute values $|b_n|$ converges to $|b|$.

(b) Is the converse of part (a) true? If we know that $|b_n| \to |b|$, can we deduce that $(b_n) \to b$?

2.3.7. (a) Let (a_n) be a bounded (not necessarily convergent) sequence, and assume $\lim b_n = 0$. Show that $\lim(a_n b_n) = 0$. Why are we not allowed to the use Algebraic Limit Theorem to prove this?

(b) Can we conclude anything about the convergence of $(a_n b_n)$ if we assume that (b_n) converges to some nonzero limit b?

(c) Use (a) to prove Theorem 2.3.3, part (iii), for the case when $a = 0$.

2.3.8. Give an example of each of the following, or state that such a request is impossible by referencing the proper theorem(s):

(a) sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges;

(b) sequences (x_n) and (y_n), where (x_n) converges, (y_n) diverges, and $(x_n + y_n)$ converges;

(c) a convergent sequence (b_n) with $b_n \neq 0$ for all n such that $(1/b_n)$ diverges;

(d) an unbounded sequence (a_n) and a convergent sequence (b_n) with $(a_n - b_n)$ bounded;

(e) two sequences (a_n) and (b_n), where $(a_n b_n)$ and (a_n) converge but (b_n) does not.

2.3.10. If $(a_n) \to 0$ and $|b_n - b| \leq a_n$, then show that $(b_n) \to b$.