Homework Assignment 9
Math 3001 — Fall 2014
Due Friday, October 24

Exercises:

4.2.1. Use Definition 4.2.1 (the definition of a functional limit) to prove that
(c) $\lim_{x\to 2} x^3 = 8$. [Hint: $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$].

4.2.8. Assume $f(x) \geq g(x)$ for all x in some set A on which f and g are defined. Show that for any limit point c of A we must have

$$\lim_{x\to c} f(x) \geq \lim_{x\to c} g(x).$$

4.2.9. Prove the Squeeze Theorem: Let f, g, and h satisfy $f(x) \leq g(x) \leq h(x)$ for all x in some common domain A. If $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$ at some limit point c of A, show that $\lim_{x\to c} g(x) = L$.

4.3.1. Let $g(x) = \sqrt[3]{x}$.
(a) Prove that g is continuous at $c = 0$.
(b) Prove that g is continuous at each point $c \neq 0$.

4.3.7. Assume $h\colon \mathbf{R} \to \mathbf{R}$ is continuous on $\mathbf{R}$, and let $K = \{x\colon h(x) = 0\}$. Show that K is a closed set.