Math 3001 — Analysis I

FALL 2014 SYLLABUS

Class Location: MUEN E064, MWF 2:00–2:50 PM
Instructor: T. Alden Gassert
Office: MATH 223
Email: Thomas.Gassert@colorado.edu
Office Hours: Th 10:00–12:00, F 10:00–11:00 (subject to change) and by appointment.
Text: Stephen Abbott, *Understanding Analysis*, Springer, 2001. ISBN-13: 978-0387950600

About the course: This course is designed to provide a rigorous understanding of the basic concepts from calculus. These topics include the construction of real line, convergence of sequences and series, the Intermediate Value Theorem, the Mean Value Theorem, and the Fundamental Theorem of Calculus. A "rigorous understanding" means that, over the course of the semester, we will develop the tools in order to prove these important results.

Course goals: The primary goal of this course is not to confirm known results, but to develop and improve mathematical intuition, understanding, and argument. By the end of this course, you should be able to write clear and precise proofs, as well as identify logic flaws in false arguments. In particular, you should keep the following in mind throughout the course:

- Your proofs should be understandable by your fellow classmates. They are your audience. They have access to the same definitions and theorems in the course, but not necessarily from beyond.
- Do not skip steps or draw upon assumptions in your proofs. This is fine when approaching a problem, but in a proof, each line should follow logically from the previous statement. Each line should be justified by simple arithmetic, a definition, or a theorem.
- A proof is composed of English sentences. The proofs in our text book, for example, are well thought out discussions on how one lays out an approach, then follows through.
- Do each problem twice. Work the problem out completely first, then put together a polished version to hand in.

I strongly encourage you to write up your homework using LaTeX, a typesetting system that is widely used in the scientific community. (To download, visit: http://www.latex-project.org/)

Grading: Your final grade for the course will be computed as follows:

20% Homework
10% Class participation
20% Midterm 1 (Monday, October 6)
20% Midterm 2 (Monday, November 10)
30% Final Exam (Wednesday, December 17, 7:30–10:00 PM)

Homework: Problem sets will be due every Friday at the beginning of class. Late homework will not be accepted, but the lowest two homework scores will be dropped when computing your final grade.

Class participation: A portion of class on Friday will be set aside for you to present your solutions in class. This is an opportunity for the class to read, discuss, and possibly improve the presented solution. Your class participation grade (out of 20 points) is based on the number of times you present and defend a solution. You are not graded on the validity of your proofs, but you may be asked to explain your thought process.

Presentations	1	2	3	4	5
Points	9	14	17	19	20

Special accommodations, religious observances, etc.: If you qualify for special accommodations due to a disability, you will need to provide to me a letter from Disability Services. Please attend to this matter in a timely fashion.

Please inform me as soon as possible if you will miss an exam or a homework assignment due to religious observance so that we have time to arrange a reasonable accommodation.

Homework Assignment 1

Exercises: 1.2.1 – 1.2.3.

1.2.1 (a) Prove that $\sqrt{3}$ is irrational. Does the same argument show that $\sqrt{6}$ is irrational?
(b) Where does the proof of Theorem 1.1.1 break down if we try to use it to prove that $\sqrt{4}$ is irrational?

1.2.2 Decide which of the following represent true statements about the nature of sets. For any that are false, provide a specific example where the statement in question does not hold.
(a) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \cdots$ are all sets containing an infinite number of elements, then the intersection $\cap_{n=1}^{\infty} A_n$ is infinite as well.
(b) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \cdots$ are all finite, nonempty sets of real numbers, then the intersection $\cap_{n=1}^{\infty} A_n$ is finite and nonempty.
(c) $A \cap (B \cup C) = (A \cap B) \cup C$.
(d) $A \cap (B \cap C) = (A \cap B) \cap C$.
(e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

1.2.3 (De Morgan's Laws.) Let A and B be subsets of $\mathbf{R}$.
(a) If $x \in (A \cap B)^c$, explain why $x \in A^c \cup B^c$. This shows that $(A \cap B)^c \subseteq A^c \cup B^c$.
(b) Prove the reverse inclusion $(A \cap B)^c \supseteq A^c \cup B^c$, and conclude that $(A \cap B)^c = A^c \cup B^c$.
(c) Show $(A \cup B)^c = A^c \cap B^c$ by demonstrating inclusion both ways.