HOMEWORK ASSIGNMENT 12
MATH 3001 — FALL 2014
DUE FRIDAY, DECEMBER 12

Exercises:

6.2.3. Consider the sequence of functions

$$h_n(x) = \frac{x}{1 + x^n}$$

over the domain $[0, \infty)$.

(a) Find the pointwise limit of (h_n) on $[0, \infty)$.
(b) Explain how we know that the convergence cannot be uniform on $[0, \infty)$.
(c) Choose a smaller set over which the convergence is uniform and supply an argument to show that this is indeed the case.

6.2.7. Assume that (f_n) converges uniformly to f on A and that each f_n is uniformly continuous on A. Prove that f is uniformly continuous on A.

6.2.9. Assume (f_n) converges uniformly to f on a compact set K, and let g be a continuous function on K satisfying $g(x) \neq 0$. Show (f_n/g) converges uniformly on K to f/g.

6.2.11. Assume (f_n) and (g_n) are uniformly convergent sequences of functions.

(a) Show that $(f_n + g_n)$ is a uniformly convergent sequence of functions.
(b) Give an example to show that the product $(f_n g_n)$ may not converge uniformly.
(c) Prove that if there exists an $M > 0$ such that $|f_n| \leq M$ and $|g_n| \leq M$ for all $n \in \mathbf{N}$, then $(f_n g_n)$ converges uniformly.