# Analysis — Spring 2015
# CU Boulder Math 3001

## worksheet 4

Read sections: 2.1–2.2

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**Definition 1.** A *sequence* is a function $f$ whose domain is $\mathbb{N}$. In other words, a sequence is an ordered list of real numbers of infinite length where the $n$-th item in the list is $f(n)$.

A sequence may be given explicitly: $(1, 3, 5, 7, \cdots)$
or by a rule: $\left(\frac{1}{n}\right)_{n=1}^{\infty} = \left(\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right)$
or the terms in the sequence may be unspecified: $(a_n)$.

**Definition 2.** A sequence $(a_n)$ *converges* to $a \in \mathbb{R}$ if for every $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that if $n \geq N$ then $|a_n - a| < \epsilon$.

We write $\lim a_n = a$ or $a_n \to a$ to indicate that the sequence converges to $a$.

**Exercise 3.** Explain, in your own words, what it means for a sequence to converge.

**Exercise 4.** Let $(a_n) = (n^{-1/2})$. Prove that $\lim a_n = 0$.

**Exercise 5.** Prove that $\lim \frac{n+1}{n} = 1$.

**Exercise 6.** Prove that $\lim \frac{3n+1}{2n+5} = \frac{3}{2}$.

**Exercise 7.** The order of the phrases in the definition of convergence is very precise; reordering the words can result in a significant change to the definition. What sequences "converge" according to the following statements?

(a) *A sequence $(a_n)$ converges to $a \in \mathbb{R}$ if there exists $\epsilon > 0$ and there exists $n \in \mathbb{N}$ such that $|a_n - a| < \epsilon$.*
(b) *A sequence $(a_n)$ converges to $a \in \mathbb{R}$ if there exists $\epsilon > 0$ such that $|a_n - a| < \epsilon$ for all $n \in \mathbb{N}$.*
(c) *A sequence $(a_n)$ converges to $a \in \mathbb{R}$ if there exists $N \in \mathbb{N}$ such that if $n \geq N$ then $|a_n - a| < \epsilon$ for all $\epsilon > 0$.*

**Exercise 8.** Using the "definitions" from the previous problem,

(a) Give an example of a sequence that converges according to definition (a) but not according to definition (b).
(b) Give an example of a sequence that converges according to definition (b) but not according to the definition of convergence (definition 2).
(c) Give an example of a sequence that converges according to the definition of convergence, but not according to definition (c).