12.6 – Cylinders and Quadric Surfaces
University of Massachusetts Amherst
Math 233 – Fall 2013

A review of the basic conic sections.

(1) **Ellipse/Circle**

(2) **Parabola**

(3) **Hyperbola**

Definition 1. A *quadric surface* is the graph of a second-degree equation in three variables x, y, and z. The general equation of a quadric surface is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.$$

However, up to translations and rotations, each surface is describable by an equation of the form

$$Ax^2 + By^2 + Cz^2 + D = 0 \quad \text{or} \quad Ax^2 + By^2 + Cz = 0.$$

Definition 2. A *cylinder* is a surface that consists of all lines (called *rulings*) that are parallel to a given plane curve.

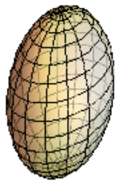
In order to sketch the graph of a surface, it is useful to determine the curves of intersection of the surface with planes parallel to the coordinate planes. These curves are called *traces* (or cross-sections) of the surface.

Example 1. Sketch and identify the cylinders defined by

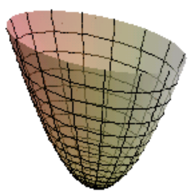
(a) $x^2 + y^2 = 1$ (b) $x^2 + 4y^2 = 4$ (c) $x^2 = z$

More quadric surfaces: (images by Wolfram|Alpha)

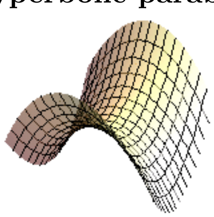
(1) **Ellipsoid/Sphere**



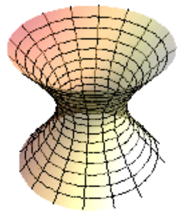
(2) **Elliptic paraboloid**



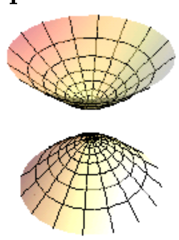
(3) **Hyperbolic paraboloid**



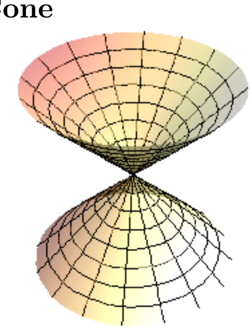
(4) **Hyperboloid of one sheet**



(5) **Hyperboloid of two sheets**



(6) **Cone**



Example 2. Sketch and identify the surfaces given by

(a) $x^2 + 9y^2 + z^2 = 4$
(b) $-3x^2 + 4y^2 = z^2$
(c) $9y^2 + z^2 = x^2 + 4$
(d) $2y^2 + z = x^2 + 1$
(e) $4y^2 - z^2 = x^2 + 1$
(f) $x^2 + 2z^2 - 6x - y + 10 = 0$