13.1 – Vector functions and space curves
University of Massachusetts Amherst
Math 233 – Fall 2013

Definition 1. A *vector-valued function*, or *vector function*, is a function whose domain is a set of real numbers, and whose range is a set of vectors. In general, we use $\vec{r}(t)$ to denote our vector valued functions. We can express $\vec{r}$ in terms of its *component functions* as follows.

$$\vec{r}(t) = \langle\, f(t), g(t), h(t)\,\rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k} \tag{1}$$

Example 1. What is the domain of $\vec{r}(t) = \left\langle\, t^3, \ln(3-t), \sqrt{t}\,\right\rangle$?

Definition 2. The *limit* of a vector function $\vec{r}$ is defined by taking the limits of its component functions. If $\vec{r}(t) = \langle\, f(t), g(t), h(t)\,\rangle$, then

$$\lim_{t\to a} \vec{r}(t) = \left\langle\, \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\,\right\rangle \tag{2}$$

Definition 3. Recall that a function $f(t)$ is *continuous* if

$$\lim_{t\to a} f(t) = f(a) \tag{3}$$

for every a in the domain of f. Similarly, we say that $\vec{r}(t)$ is *continuous* if

$$\lim_{t\to a} \vec{r}(t) = r(a) \tag{4}$$

for every a in the domain of $\vec{r}(t)$.

Exercise 1. *Let $\vec{r}(t) = \langle\, f(t), g(t), h(t)\,\rangle$. Prove that $r(t)$ is continuous if and only if $f(t)$, $g(t)$, and $h(t)$ are all continuous.*

Example 2. Determine $\lim_{t\to 0} \vec{r}(t)$ when $\vec{r}(t) = \left\langle\, 1+t^3, te^{-t}, \frac{\sin t}{t}\,\right\rangle$.

Definition 4. Let $f(t)$, $g(t)$, and $h(t)$ be continuous, real-valued functions and $\vec{r}(t) = \langle\, f(t), g(t), h(t)\,\rangle$. The set of points (x, y, z) given by

$$x = f(t), \quad y = g(t), \quad z = h(t) \tag{5}$$

as t varies through the domain of $\vec{r}$ defines a *space curve* C. The equations in (5) are *parametric equations* for C, and t is a *parameter*.

Example 3. Describe the curve defined by $\vec{r}(t) = \langle\, 1 + t, 2 + 5t, -1 + 6t\,\rangle$.

Example 4. Sketch the curve defined by $\vec{r}(t) = \langle\, \cos t, \sin t, t\,\rangle$.

Example 5. Find a vector equation and parametric equations for the line segment that joins the point $P(1,3,-2)$ and $Q(2,-1,3)$.

Example 6. Find a vector function that represents the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $x+z=2$.

Example 7. Find a vector function that represents the curve of intersection of the cylinder $x^2+y^2=2$ and the surface $z=1-x^2y^2$.