13.4 – Motion in space: velocity and acceleration
University of Massachusetts Amherst
Math 233 – Fall 2013

Definition 1. If $\vec{r}(t)$ is the position vector of an object at time t, its *velocity vector* $\vec{v}(t)$ at time t is

$$\vec{v}(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \vec{r}\,'(t)$$

The *speed* of the object at time t is the magnitude of the velocity vector, $|\vec{v}(t)|$. Its acceleration is

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$$

Example 1. Find the velocity, acceleration, and speed of a particle with position vector $\vec{r}(t) = \langle \cos t, \sin t, te^t \rangle$.

Solution.

Note. If we know initial position and velocity, we can find

$$\vec{v}(t) = \vec{v}(t_0) + \int_{t_0}^{t} \vec{a}(u)\,du \qquad\qquad \vec{r}(t) = \vec{r}(t_0) + \int_{t_0}^{t} \vec{v}(u)\,du$$

Example 2. A projectile is fired with angle of elevation α and initial velocity $\vec{v}_0$. Assuming air resistance is negligible, and the only external force is due to gravity, find the position function $\vec{r}(t)$ of the projectile. What value of α maximizes the range (the horizontal distance traveled)?

Solution.

Example 3. A projectile is fired with muzzle speed 160 m/s and angle of elevation 30° from a position 10 m above ground level. Where does the projectile hit the ground, and with what speed?

Solution.