14.1 – Functions of several variables University of Massachusetts Amherst Math 233 – Fall 2013

**Definition 1.** A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by f(x, y). The set D is the domain of f, and its range is the set of value that f takes, i.e.

$$\{f(x,y)\colon (x,y)\in D\}.$$

We often write z = f(x, y), which is analogous to the single variable case where we write y = f(x). Example 1. Determine the domain and range of  $f(x, y) = \sqrt{9 - x^2 - y^2}$ , and evaluate f(-1, 2).

**Definition 2.** If f is a function of two variables, the graph of f is the set of all points (x, y, z) satisfying z = f(x, y).

**Example 2.** Sketch the graph of the function  $f(x, y) = \sqrt{9 - x^2 - y^2}$ 

**Example 3.** Sketch the graph of the function f(x, y) = 6 - 3x - 2y.

**Definition 3.** The *level curves*, or *contour lines*, of a function f of two variables are the curves with the equations f(x, y) = k, where k is a constant.

**Example 4.** Sketch the level curves for the function  $f(x,y) = \sqrt{9 - x^2 - y^2}$  for k = 0, 1, 2, 3.

What does the distance between contour lines say about the graph?





2 -2

**Example 5.** Match the contour diagrams to their graphs.