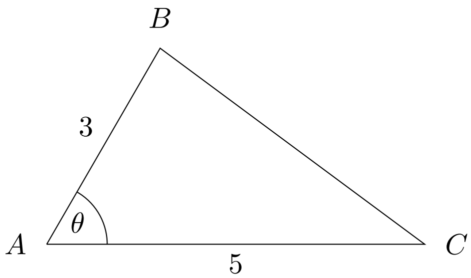
QUIZ 2
University of Massachusetts Amherst
Math 233 – Fall 2013
September 20, 2013

NAME:

Consider two vectors $\vec{u}$ and $\vec{v}$ satisfying $|\vec{u}| = 3$ and $|\vec{v}| = 5$. (Two points per problem.)

(1) If $\vec{u}$ is perpendicular to $\vec{v}$, then $\vec{u} \cdot \vec{v} =$ ____________.
(2) If $\vec{u}$ is parallel to $\vec{v}$, then $\vec{u} \cdot \vec{v} =$ ____________.
(3) Determine the area of the triangle $\triangle ABC$ if $\theta = \pi/3$.



(4) Determine $\vec{u} \cdot (\vec{u} \times \vec{v})$.

SOLUTIONS: Questions (1) and (2) follow from the relation

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta,$$

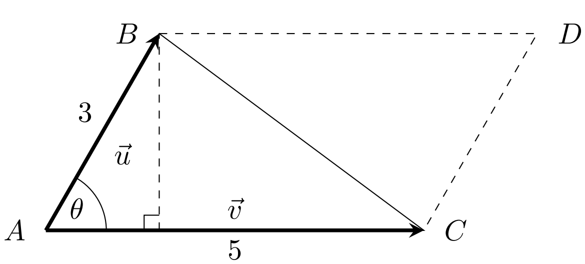
where θ is the angle between $\vec{u}$ and $\vec{v}$.

(1) ANSWER: If $\vec{u}$ and $\vec{v}$ are perpendicular, then $\theta = \pi/2$, hence $\boxed{\vec{u} \cdot \vec{v} = 0}$.

(2) ANSWER: If $\vec{u}$ is parallel to $\vec{v}$, then $\theta = 0$, hence $\boxed{\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| = 15}$.

(3) ANSWER: Recall that $|\vec{u} \times \vec{v}|$ is the area of the parallelogram $ABDC$ generated by the vectors $\vec{u}$ and $\vec{v}$. Moreover, $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$. The area of the triangle is half the area of the parallelogram. Thus the area of the triangle is

$$\frac{|\vec{u}||\vec{v}| \sin\theta}{2} = \frac{15}{2} \sin(\pi/3) = \boxed{\frac{15\sqrt{3}}{4}}.$$



Alternatively, the area of the triangle is $bh/2$, where the base of the triangle is 5 and the height of the triangle is $3\sin(\pi/3)$. Once again, we find that the area is $\left(15\sqrt{3}\right)/4$.

(4) ANSWER: The triple produce is $\boxed{0}$ because the vectors $\vec{u}$, $\vec{u}$, and $\vec{v}$ are coplanar. A similar argument is that the vector $\vec{u} \times \vec{v}$ is perpendicular to $\vec{u}$, hence $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.