QUIZ 4
UNIVERSITY OF MASSACHUSETTS AMHERST
MATH 233 – FALL 2013
OCTOBER 14, 2013

NAME: 2 points per problem

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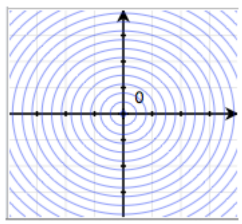
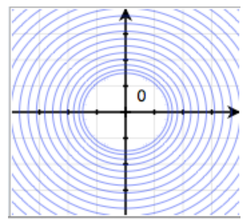
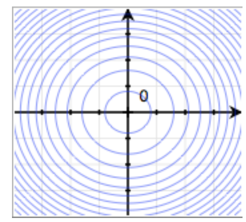
(1) Which of the following contour diagrams corresponds to the cone $z^2 = x^2 + y^2$.

a) 

b) 

c) 

Explain your choice. (+1 point each for naming the quadric surfaces associated to the other diagrams: ellipsoid, hyperboloid, elliptic paraboloid, hyperbolic paraboloid)

ANSWER: The surface of the cone has constant slope when traveling radially away from the origin. Hence the contours on the cone are evenly spaced. The correct diagram is $\boxed{\text{(a)}}$. Diagram (b) corresponds to a hyperboloid of one sheet. The surface is steepest near the origin, and less steep as one moves away from the origin. Diagram (c) corresponds to a paraboloid. The steepness of the paraboloid increases further from the origin.

(2) Show that the following limit does not exist. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$

ANSWER: It is simplest to take $(x, y) \to (0, 0)$ along $x = 0$ and $y = 0$. When $x = 0$, the limit becomes

$$\lim_{y\to 0} \frac{-y}{y} = \lim_{y\to 0} -1 = -1,$$

and along $y = 0$, the limit is

$$\lim_{x\to 0} \frac{x}{x} = \lim_{x\to 0} 1 = 1.$$

These two limits are unequal, hence the limit as $(x, y) \to (0, 0)$ does not exist.

Approaching $(0, 0)$ along any two lines (except $y = -x$, where the function is undefined) will work. Let $y = mx$. Then

$$\lim_{(x,mx)\to(0,0)} \frac{x-mx}{x+mx} = \lim_{x\to 0} \frac{x(1-m)}{x(1+m)} = \lim_{x\to 0} \frac{1-m}{1+m} = \frac{1-m}{1+m}.$$

Thus the value of the limit depends on the slope m.

(3) Consider the function $f(x,y) = y^3e^x + 1$.
  (a) Compute $f_x(x,y)$ and $f_y(x,y)$.
  (b) Evaluate $f_x(0,-2)$ and $f_y(0,-2)$.
  (c) Let $\vec{v}$ be the vector $\langle f_x(0,-2), f_y(0,-2)\rangle$. What is the length of $\vec{v}$?

[Hints: (i) $\vec{v}$ is orthogonal to $\langle 3,2\rangle$. (ii) The length of a vetoer in two dimensions may be computing using the pythagorean theorem.]

ANSWER:
(a) $f_x(x,y) = y^3e^x$, $f_y(x,y) = 3y^2e^x$.
(b) $f_x(0,-2) = -8$, $f_y(0,-2) = 12$.
(c) $|\vec{v}\,| = \sqrt{8^2+12^2} = \sqrt{208}$.
We can check that we are on the right track using the hint that $\vec{v}$ is orthogonal to $\langle 3,2\rangle$. That is, $\vec{v}$ must satisfy $\vec{v}\cdot\langle 3,2\rangle = 0$. A quick calculation shows that this equality holds: $\langle -8,12\rangle\cdot\langle 3,2\rangle = -24+24 = 0$. This don't necessarily tell us that we've computed $\vec{v}$ correctly. But, if your $\vec{v}$ does not satisfy this dot product, then you've certainly made a mistake.

(4) Use *implicit differentiation* to compute $\dfrac{\partial z}{\partial x}$ for the equation $z^4 = \sin(xy)$.

ANSWER: Taking partial derivatives on both sides, we find

$$\begin{aligned}\frac{\partial}{\partial x}z^4 &= \frac{\partial}{\partial x}\sin(xy)\\ 4z^3\frac{\partial z}{\partial x} &= y\cos(xy)\\ \frac{\partial z}{\partial x} &= \frac{y\cos(xy)}{4z^3}.\end{aligned}$$